# NATIONAL ADVISORY COMMITTEE
# FOR AERONAUTICS

## TECHNICAL NOTE 3471

THEORETICAL ANALYSES TO DETERMINE

UNBALANCED TRAILING-EDGE CONTROLS HAVING MINIMUM HINGE

MOMENTS DUE TO DEFLECTION AT SUPERSONIC SPEEDS

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SUMMARY

Analyses based on theoretical results of NACA Report 1041 have been made to determine the plan forms of unbalanced trailing-edge flap-type controls having minimum hinge moments due to deflection and requiring minimum work to overcome the hinge moments due to deflection at supersonic speeds. Ratios of lift and rolling moment to hinge moment and ratios of lift and rolling moment to deflection work at fixed values of lift and rolling effectiveness were used as bases for the analyses.

Results of the analyses for longitudinal controls show high-aspect-ratio untapered controls to possess maximum ratios of lift $L$ to hinge moment $H$. When low-aspect-ratio controls must be used, however, controls with triangular plan forms and highly swept hinge lines are shown to have higher values of $L/H$ than untapered controls. Ratios of lift to deflection work for untapered controls are in most cases shown to be higher than those for controls with tapered plan forms.

On wings with sweptforward and unswept trailing edges, inversely tapered controls with triangular plan forms of moderate or low aspect ratio are shown to have maximum ratios of rolling moment $L'$ to hinge moment $H$. On wings with sweptback trailing edges, maximum values of $L'/H$ are shown for either untapered or normally tapered controls. For any given control shape, the analysis illustrates the importance of using small controls with high deflections to obtain large values of $L'/H$.

Maximum ratios of rolling moment to deflection work on wings with sweptforward trailing edges are in most cases obtained with inversely tapered controls with triangular plan forms. On wings with unswept and sweptback trailing edges, the deflection work required is near minimum for untapered controls with spans of about two-thirds the wing semispan. Results indicate that large controls will in most cases have higher ratios of rolling moment to deflection work than smaller controls.

[1]Supersedes recently declassified NACA RM L51F19, 1952.

INTRODUCTION

The control forces on aircraft operating at supersonic speeds are
so high that very substantial power-boost systems are usually required
to handle the hinge moments. As an approach to a solution of the prob-
lem of reducing the size and work requirements of boost systems for such
aircraft, theoretical analyses have been made of the hinge moments due
to deflection of unbalanced trailing-edge flap-type controls with plan
forms varying throughout the range in which the control leading and
trailing edges are supersonic and the control tips are streamwise. Aero-
elastic effects are not included and the analyses, which are based on
equations and charts from reference 1, are subject to the limitations of
linearized theory.

When the analyses were made, values of lift and rolling-moment coef-
ficients and parameters indicative of ratios of lift and rolling moment
to hinge moment $\left(F_L \text{ and } F_l\right)$ were calculated for a range of control
plan forms on wings having various trailing-edge sweep angles. (Trailing-
edge sweep angle was the only wing-plan-form parameter which had to be
specified because the loading over the portion of a wing ahead of a con-
trol is not influenced by control deflection at supersonic speeds.) From
these calculations, families of controls having fixed amounts of effec-
tiveness were determined, and the corresponding parameters $F_L$ and $F_l$
were plotted as functions of the various control plan-form parameters.
From the resulting charts, the plan forms of controls producing fixed
amounts of lift and rolling moment with minimum hinge moments due to
deflection were determined. Similar analyses were also made to determine
the plan forms of controls requiring minimum amounts of work to overcome
the hinge moments due to deflection. (The analyses for deflection work
are similar to analyses carried out by Jones and Cohen for the incom-
pressible case and presented in ref. 2.) The hinge-moment analyses are
applicable in cases where the strength of the actuating mechanism or the
amount of torque available at the control is the design criteria. The
work analyses are applicable when the design criteria are the energy which
must be carried for operating the boost system or the energy which the
pilot must exert in event of boost failure.

Hinge moments due to angle of attack, damping in pitch, and rolling
depend on the wing plan form and to varying degrees on the complete air-
craft configuration and have not been included in the present analyses
because the calculations involved would have been exorbitant. The fol-
lowing comments regarding these neglected hinge moments should therefore
be kept in mind when the results of the analyses are applied: Hinge
moments due to angle of attack and damping in pitch are of primary impor-
tance with regard to longitudinal controls because hinge moments of such
controls are equal to the algebraic summation of these hinge moments and

the hinge moments due to deflection. Consequently, complete analyses for longitudinal controls would require, in addition to the analyses of the present paper, similar analyses in which hinge moments due to angle of attack and damping in pitch are considered. With regard to the combined hinge moments of differentially deflected lateral controls, hinge moments due to angle of attack and damping in pitch are of no significance because the effects on opposite ailerons cancel. It is possible, however, that, in some cases, the hinge moments of the individual ailerons are of more importance than their combined hinge moments, for instance, when the ailerons are not interconnected but are actuated independently. In such cases, hinge moments due to angle of attack and damping in pitch would have to be considered. Hinge moments due to rolling are of primary importance with regard to lateral controls because in most cases they tend to reduce the hinge moments due to deflection of ailerons on both wing panels. It is estimated that, for the unbalanced trailing-edge flap-type controls considered in the present paper, hinge moments due to rolling in the most critical cases are not likely to reduce hinge moments due to deflection by more than 15 or 20 percent. Hinge moments of this order are certainly of importance with regard to the actuation of controls but are probably of minor importance with regard to the selection of low-hinge-moment controls. The hinge moments of longitudinal controls due to rolling are probably of less significance because the controls are usually located considerably nearer the axis of symmetry than lateral controls and, consequently, in regions where the induced angles of attack due to rolling are smaller.

## SYMBOLS

| | |
|---|---|
| M | free-stream Mach number |

$$\beta = \sqrt{M^2 - 1}$$

| | |
|---|---|
| q | free-stream dynamic pressure |
| $\Lambda$ | angle of sweep of wing leading edge, positive when swept back, deg |
| $\Lambda_{HL}$ | angle of sweep of control hinge line, positive when swept back, deg |
| $\Lambda_{TE}$ | angle of sweep of wing trailing edge, positive when swept back, deg |
| $\delta$ | angle of control-surface deflection measured in streamwise direction, deg |

$\delta_1$          maximum value of $\delta$

$$a = \frac{\tan \Lambda_{HL}}{\beta}$$

$b$           wing span, ft

$A$           wing aspect ratio

$\lambda$           wing taper ratio

$S$           wing area

$y_f$          distance from wing root chord to inboard parting line
             of control

$b_f$          control span

$c_t$          tip chord of control

$c_r$          root chord of control

$\lambda_f$          control-surface taper ratio, $c_t/c_r$

$S_f$          area of control surface

$A_f$          aspect ratio of control surface, $b_f^2/S_f$

$M_a$          area moment of control surface about hinge axis

$L$           lift induced by control deflection

$L'$          moment about wing root chord induced by control deflection

$H$           hinge moment due to control deflection

$H_{(p)}$        hinge moment due to rolling

$W$           work required to overcome hinge moments due to control
             deflection (deflection work)

$$F_L = \beta b \cos \Lambda_{HL} \frac{L}{H} = \frac{\delta_1}{57.3} \frac{\beta b}{2} \frac{L}{W}$$

$$F_l = \beta \cos \Lambda_{HL} \frac{L'}{H} = \frac{\delta_1}{57.3} \frac{\beta}{2} \frac{L'}{W}$$

$$C_L = \frac{L}{qS}$$

$$C_l = \frac{L'}{qbS}$$

$$C_h = \frac{H}{2M_a q}$$

$p$          rolling rate, radians/sec

$V$          velocity, ft/sec

$\frac{pb}{2V}$          wing-tip helix angle, radians

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

Subscript:

$\delta$          partial derivative of force and moment coefficients
            with respect to   $\delta$

## ANALYSIS

### General

The regions in which loading is influenced by the deflection of trailing-edge controls at supersonic speeds are limited to the surfaces of the deflected controls and to portions of the wings adjacent to the controls and lying within the Mach cones from the control tips (fig. 1). Within the scope of the present paper, the loadings induced by deflected controls are unaffected by wing plan form; therefore, the trailing-edge sweep angle, which defines the regions of induced loading on the wing, is the only wing parameter necessary for determining the characteristics of trailing-edge controls. It was convenient to choose various values of $\frac{\tan \Lambda_{TE}}{\beta}$ and, for each of these values, to vary systematically the control plan form and location. The analyses, with fixed Mach numbers

assumed, then correspond to examinations of the effects of control plan form and location on the characteristics of control surfaces on wings having various trailing-edge sweep angles.

All calculations were made for controls located on left wing panels so that positive (downward) control deflections would result in positive rolling moments as well as positive lift. Since positive control deflections in all cases result in negative hinge moments, the ratios of lift and rolling moment to hinge moment are negative, and the functions of these ratios, $F_L$ and $F_l$, presented in the charts, are also negative. These ratios and functions are discussed throughout the report in terms of absolute magnitude; that is, the most negative values are referred to as maximum.

The parameters used as bases for comparison in the analyses for determining maximum ratios of lift and rolling moment to hinge moment were $F_L = \beta b \cos \Lambda_{HL}\left(\dfrac{L}{H}\right)$ and $F_l = \beta \cos \Lambda_{HL}\left(\dfrac{L'}{H}\right)$. The $\beta$ and $\cos \Lambda_{HL}$ terms were included in the parameters $F_L$ and $F_l$ in order to avoid considering Mach number as an independent variable in the calculations. When these parameters are used, Mach number enters the calculations only as part of the plan-form parameters $\dfrac{\tan \Lambda_{HL}}{\beta}$, $\dfrac{\tan \Lambda_{TE}}{\beta}$, and $\beta A_f$ (for untapered plan forms); consequently, for any given control plan form, variations in Mach number correspond to variations in $\dfrac{\tan \Lambda_{HL}}{\beta}$, $\dfrac{\tan \Lambda_{TE}}{\beta}$, and/or $\beta A_f$. A linear dimension was needed to make the parameter $F_L$ nondimensional and the term $b$ was included for this purpose.

When the parameters $F_L$ and $F_l$ are used as bases for comparison in determining the plan forms and locations of controls having maximum ratios of lift and rolling moment to hinge moment, the effects of the term $\cos \Lambda_{HL}$ in $F_L$ and $F_l$ must be considered. Because of the $\cos \Lambda_{HL}$ term, values of $F_L$, for instance, for one value of $\Lambda_{HL}$ may possibly be higher than values for some higher absolute value of $\Lambda_{HL}$; whereas the value of $L/H$ for the lower value of $\Lambda_{HL}$ is less. This fact can be illustrated as follows:

$$\left(\frac{L}{H}\right)_2 = \frac{\left(\beta b \cos \Lambda_{HL}\right)_1 F_{L_2}}{\left(\beta b \cos \Lambda_{HL}\right)_2 F_{L_1}}\left(\frac{L}{H}\right)_1 \tag{1}$$

where the subscript 1 refers to conditions corresponding to some arbitrary value of $\Lambda_{HL}$ and the subscript 2 refers to some other arbitrary value of $\Lambda_{HL}$. In order to make proper comparisons of various control plan forms, a fixed Mach number and a fixed wing plan form must be assumed. In such cases, values of $\beta$ and $b$ are constant and equation (1) reduces to

$$\left(\frac{L}{H}\right)_2 = \frac{\cos \Lambda_{HL_1} F_{L_2}}{\cos \Lambda_{HL_2} F_{L_1}}\left(\frac{L}{H}\right)_1 \tag{1a}$$

Although $F_{L_2}$ may be less than $F_{L_1}$ depending on the ratios of the functions and the $\cos \Lambda_{HL}$ terms, $\left(\frac{L}{H}\right)_2$ may possibly be greater than $\left(\frac{L}{H}\right)_1$. Inasmuch as the hinge-line sweep parameter used in defining control plan form in the present paper is $a = \dfrac{\tan \Lambda_{HL}}{\beta}$, it is convenient to rewrite equation (1a) in terms of $a$:

$$\left(\frac{L}{H}\right)_2 = \left(\frac{L}{H}\right)_1 \frac{F_{L_2}}{F_{L_1}}\sqrt{\frac{1 + \beta^2 a_2{}^2}{1 + \beta^2 a_1{}^2}} \tag{1b}$$

The parameters $F_L$ and $F_l$ are also convenient for use in the analyses of controls on the basis of minimum deflection work. This can be shown as follows:

$$W = \int_0^{s_1} F\ ds \tag{2}$$

where the subscript 1 denotes maximum displacement and

F        force on control, $\dfrac{H}{\bar{x}} = \dfrac{2M_a q \delta C_{h_\delta}}{\bar{x}}$

s        deflection of point on control at which center of loading lies, $\dfrac{\bar{x}\delta}{57.3 \cos \Lambda_{HL}}$

$\bar{x}$            distance from hinge axis to center of loading on control
measured normal to hinge axis

Rewriting equation (2) yields

$$W = \frac{2M_a q C_{h_\delta}}{57.3 \cos \Lambda_{HL}} \int_0^{\delta_1} \delta \; d\delta \qquad (2a)$$

Integrating and reducing equation (2a) yields

$$W = \frac{\delta_1}{57.3} \frac{H_1}{2 \cos \Lambda_{HL}} \qquad (2b)$$

Rearranging the terms of equation (2b) gives

$$H_1 = \frac{2 \cos \Lambda_{HL}}{\dfrac{\delta_1}{57.3}} W \qquad (3)$$

When the value of hinge moment $H_1$ from equation (3) is substituted into
the equations $F_L = \beta b \cos \Lambda_{HL}\left(\dfrac{L}{H}\right)$ and $F_l = \beta \cos \Lambda_{HL}\left(\dfrac{L'}{H}\right)$, the param-
eters $F_L$ and $F_l$ in terms of deflection work become $F_L = \dfrac{\delta_1}{57.3} \dfrac{\beta b}{2} \dfrac{L}{W}$

and $F_l = \dfrac{\delta_1}{57.3} \dfrac{\beta}{2} \dfrac{L'}{W}$. From these definitions it can be seen that, in cases
where comparisons are made of controls at equal deflections, maximum
values of $F_L$ and $F_l$ correspond to maximum values of $L/W$ and $L'/W$,
$\beta$ and $b$ being assumed fixed. When comparisons are made of controls
at different deflections, however, this condition is not necessarily
true and the effects of the $\delta_1$ term in $F_L$ and $F_l$ have to be taken
into account.

## Longitudinal Controls

In the analyses for longitudinal controls, controls located at the
inboard, midspan, and tip positions on the wings were included. Figure 2
illustrates these positions together with the limiting Mach line locations

for each position. As shown in figure 2 for tip controls considered in the present paper, the Mach lines from the control root chords did not cross either the wing root chords or the wing tips. For midspan controls, the Mach lines from the controls did not cross the wing root chords or the wing tips, and the Mach lines from the control root chords did not cross the control tips. For inboard controls, the Mach lines from the control tips did not cross either the wing root or wing tip chords, and the Mach lines from the control root chords did not cross the control tips. The present paper includes results for controls having root chords coincident with the wing root chords, whereas the data presented in reference 1 are limited to controls for which the innermost Mach lines do not cross the wing root chords. In order to obtain the characteristics of controls located adjacent to the wing root chords, reflection planes, which would be expected to approximate the effects of fuselages in practice, were assumed to be located at the wing root chords; loading parameters for the inboard conical flow regions of these controls were obtained from figure 7 of reference 1.

In the analyses, a range of control shape and size capable of producing a fixed lift-coefficient slope of $\dfrac{C_{L_\delta}}{A} = 0.0001$ was determined for each control position. Values of the parameter $F_L$ were calculated for these controls and are presented in figure 3 as functions of the various control plan-form parameters. The sketches at the right of the charts illustrate the hinge-line and trailing-edge sweep angles corresponding to the various curves in the accompanying charts when $\beta$ is equal to 1.0 ($M = \sqrt{2}$) and are intended only as an aid in orienting the reader.

Although the value of the lift-coefficient slope used in the calculations for the charts of figure 3 $\left(\dfrac{C_{L_\delta}}{A} = 0.0001\right)$ is quite arbitrary, these charts have a wide range of application because, for a given control shape and wing, the value of $C_{L_\delta}/A$ is directly proportional to the square of the control span and the value of $L/H$ is inversely proportional to the control span. By use of such proportions, the following equations for extending the data of figure 3 to include other values of lift-coefficient slope are simply derived:

$$\left(\frac{2b_f}{b}\right)_1 = \left(\frac{2b_f}{b}\right)_0 \sqrt{\frac{\left(C_{L_\delta}/A\right)_1}{\left(C_{L_\delta}/A\right)_0}} \tag{4}$$

$$\left(\frac{L}{H}\right)_1 = \left(\frac{L}{H}\right)_0 \sqrt{\frac{\left(C_{L_\delta}/A\right)_0}{\left(C_{L_\delta}/A\right)_1}} \qquad\qquad (5)$$

The subscript $0$ refers to conditions when $\dfrac{C_{L_\delta}}{A} = 0.0001$ and the sub-
script $1$ refers to similar conditions for other arbitrary values
of $C_{L_\delta}/A$.

From equations $(3)$ and $(5)$, the equation for the ratio of lift to
deflection work at values of $C_{L_\delta}/A$ other than $0.0001$ becomes

$$\left(\frac{L}{W}\right)_1 = \left(\frac{L}{W}\right)_0 \frac{\left(\delta_1\right)_0}{\left(\delta_1\right)_1} \sqrt{\frac{\left(C_{L_\delta}/A\right)_0}{\left(C_{L_\delta}/A\right)_1}} \qquad\qquad (6)$$

## Lateral Controls

Limitations of analyses.- In order to obtain some indication of the
limitations of the analyses for lateral controls resulting from the
neglect of hinge moments due to rolling motions, sample calculations have
been made for the steady-roll condition in which the wing damping moment
is equal in magnitude to the rolling moment induced by aileron deflection.

Figure 4 presents theoretical ratios of hinge moment due to rolling
to hinge moment due to aileron deflection, calculated by use of equations
from references 3 and 4, for 60° delta wings with aileron controls com-
prising various amounts of the wing tips. These configurations were chosen
because the theoretical unit damping forces on such ailerons are unusually
high (especially when the control spans are relatively small and the wing
leading edges are subsonic), and hinge moments of such ailerons due to
damping therefore approach maximum values. For the configurations to be
applicable in the present analysis (for unbalanced trailing-edge flap-type
controls), it was necessary to assume the ailerons to be hinged about
their leading edges, which coincide with the wing leading edges. Although
these particular configurations are not of practical interest, they prob-
ably give a reasonable indication of the maximum hinge moments due to
rolling moment which might be obtained.

The data of figure 4 indicate that hinge moments due to rolling are
sizable and at first increase rapidly with control size. The rate of
increase diminishes as the control size is increased, however, and the
data appear to indicate that, for extremely large controls, the ratio of
hinge moment due to rolling to hinge moment due to aileron deflection
approaches a value equal to or slightly greater than 0.5. For controls

comprising 10 to 15 percent of the wing area, which might be considered
to be near the upper limit of the practical range for this type of con-
trol, it is shown that hinge moments due to rolling cancel out about
one-third the hinge moment due to deflection. Inasmuch as the data of
figure 4 are for the steady-roll condition, this value of one-third is
probably a great deal higher than that which could be expected in prac-
tice. Because of aircraft inertia, the rate of roll at the time the
control reaches maximum deflection is considerably less than the steady-
roll rate. On the basis of time histories presented in reference 5, a
rate of one-half the steady-roll rate would seem to be more nearly of
the right order, in which case the hinge moment due to rolling would
balance out only about one-sixth the hinge moment due to deflection. It
would thus appear that the analysis would not be seriously limited
because the hinge moments due to rolling were neglected. Although com-
parisons of ratios of rolling moment to hinge moment due only to deflec-
tion might in some cases result in erroneous conclusions regarding the
more desirable control, this happens only when the ratios $L'/H$ for the
controls being compared are nearly equal. It should be remembered that
the present analysis considers only unbalanced trailing-edge flap-type
controls and that all-movable or balanced flap-type controls would require
an entirely different analysis.

Method of analysis.- In the analyses for lateral controls it was
not possible to treat control size and control location in the general
manner used for longitudinal controls; consequently, the analyses are
considerably more detailed than were those for longitudinal controls.

It would seem probable that controls located at the wing tips would
in all cases have higher ratios of rolling moment to hinge moment and
rolling moment to deflection work because of their greater distance from
the roll axis. Lift and hinge moment vary with location, however, and
it is therefore necessary to determine whether this is true. In order
to do this, the effects of spanwise location on the values of $F_l$ have
been calculated for a systematic range of control plan forms, and results
of these calculations are presented in figure 5 where $F_l$ is plotted
against $2y_f/b$. The range of plan form considered is indicated by the
sketches at the right of the charts in figure 5 where the hinge-line and
trailing-edge sweep angles are shown for $\beta = 1.0$ $(M = \sqrt{2})$. The most
inboard control locations for which results are presented in figure 5
are such that the innermost Mach lines from the controls pass through
the points of intersection of the wing root chords and wing trailing
edges. The most outboard locations are such that the tip chords of the
controls and wings are coincident, as shown in figure 2 for longitudinal
controls. An examination of figure 5 reveals that in most cases controls
located at the wing tips have higher (more negative) values of $F_l$ than
do the same controls when located farther inboard. In the few cases for
which this is not true (on wings having sweptback trailing edges,
figs. 5(d) and 5(e)), the advantages of slightly inboard locations are

not large; thus, it would be sufficient in the present analysis to consider only controls located at the wing tips. It must be cautioned, however, that tip controls on wings with sweptback trailing edges will in some cases have considerably less effectiveness than controls located farther inboard, particularly in the transonic speed range (refs. 6 and 7). It should also be pointed out that, for some wing configurations, aeroelastic and viscous effects, which have not been considered in this analysis, might outweigh the advantages of tip location for the controls.

Figure 6 presents the results of calculations made to determine the values of $C_{l\delta}/A$ and $F_l$ for a range of control plan forms located at the tips of wings having various ratios of $\dfrac{\tan \Lambda_{TE}}{\beta}$. By use of the data presented in figure 6, it was possible to prepare the charts in figure 7 to show the variation of the parameter $F_l$ with plan-form parameters for controls which produce various fixed amounts of rolling moment. Values of $\dfrac{C_{l\delta}}{A}$ of 0.0002, 0.0004, and 0.0006 were chosen as representative. As in figures 3, 5, and 6, the sketches at the right of the charts illustrate the hinge-line and trailing-edge sweep angles in the accompanying charts when $\beta$ is equal to 1.0 ($M = \sqrt{2}$). It should be pointed out that, although tip chord was used to define control plan form in figures 5 and 6 for reasons of convenience in the necessary computations, aspect ratio has been used in the analysis charts of figure 7 because of its greater significance.

## RESULTS AND DISCUSSION

The discussions of the analysis charts for longitudinal and lateral controls (figs. 3 and 7) are each divided into two parts. The first part of the discussion for longitudinal controls deals with controls having maximum ratios of lift to hinge moment, and the second part deals with controls having maximum ratios of lift to deflection work. The division for lateral controls is similar. The results of the analysis are summarized in table I.

### Ratios of Lift to Hinge Moment

General.- It will be noticed that little data are presented in figure 3 for low-aspect-ratio inboard controls and for low-aspect-ratio midspan and tip controls on wings having sweptback trailing edges. This lack of data results from the limiting Mach line locations which are shown in figure 2 and have been previously discussed.

Effects of spanwise location.- From a comparison of the curves in the charts for the inboard position with those for the midspan position, it can be seen that in no case does an inboard control have a greater value of $F_L$ than does a midspan control having the same plan form. This result might be expected since inboard controls have been assumed to be located adjacent to reflection planes, and any portion of the loading normally carried by the adjacent wing which is reflected back onto the control would increase the hinge moment and probably result in lower values of L/H. It should be pointed out, however, that for high-aspect-ratio untapered controls and for inversely tapered controls having small root chords, the adverse effects of the reflection planes are not large.

In the charts for the midspan and tip-control positions (fig. 3), it will be noticed that, if values of $\beta A_f$ less than 1.0 (which seem impractically small) are neglected, the maximum value of $F_L$ shown on each curve occurs at the maximum value of $\beta A_f$. From a comparison of the curves for the midspan and tip positions, it can be seen that values of $F_L$ at the maximum values of $\beta A_f$, on corresponding curves, are in all cases for the midspan position equal to or higher than those for the tip position.

One other general group of controls which should be discussed is full-span controls. The loading of a full-span control having any particular shape would be obtained by assuming a reflection plane to be located adjacent to the root chord of a tip control having the same plan form and making a corresponding correction to the loading of the tip control. Since comparisons of inboard and midspan control positions have indicated that reflection planes, if having any effect, decrease the values of L/H, full-span controls would be expected to have values of L/H equal to or less than those for tip controls.

It thus appears that values of L/H for midspan controls will always be equal to or higher than those for similar controls at other locations. Consequently, the analysis for determining the plan forms of controls having maximum values of L/H will be limited to the consideration of controls located at midspan positions.

Untapered controls.- The charts for midspan controls show maximum values of $F_L$ in most cases for untapered controls having values of $\beta A_f$ equal to 8.0 (the upper limit of the calculations). The curves for untapered controls, if extended to higher aspect ratios, would be expected to show still higher values of $F_L$ because the lift has been fixed, and higher aspect-ratio controls would necessarily have smaller chords and, consequently, smaller moment arms and hinge moments. It

therefore appears that maximum values of L/H are obtained by use of very high-aspect-ratio untapered controls. In practice, however, it is not in most cases possible to obtain sufficient lift by use of extremely high-aspect-ratio controls. When the lift requirements are sufficiently high to require the use of moderate- and low-aspect-ratio controls, the data in figure 3 show that untapered controls will probably not have maximum values of L/H.

Tapered controls.- In the charts of figure 3, the maximum aspect ratios shown for tapered controls, represented by points farthest to the right, are the maximum aspect ratios possible for the particular combination of hinge-line and trailing-edge sweep and, consequently, represent triangular control plan forms. The only exceptions are the curves for a = 0.80 in figure 3(e), where the aspect ratio corresponding to triangular controls is beyond the range of the calculation.

It should be pointed out, as previously mentioned, that in comparing controls having various hinge-line sweep angles to determine which sweep angle gives the maximum values of L/H, comparisons must be made on the basis of $\dfrac{F_L}{\cos \Lambda_{HL}}$ rather than simply $F_L$ as plotted in figure 3. When a, that is, $\dfrac{\tan \Lambda_{HL}}{\beta}$, is equal to zero, $\cos \Lambda_{HL}$ is equal to 1.0 and $\dfrac{F_L}{\cos \Lambda_{HL}}$ is equal to $F_L$. With increases in the absolute value of a, however, $\cos \Lambda_{HL}$ decreases and $\dfrac{F_L}{\cos \Lambda_{HL}}$ increases. Consequently, comparisons on the basis of L/H must be made by shifting the curves for finite values of a downward, the amount to depend on the value of a and Mach number since $\cos \Lambda_{HL} = \dfrac{1}{\sqrt{1 + \beta^2 a^2}}$.

The charts for the midspan control positions in figures 3(a) and 3(b) show that tapered controls having maximum values of L/H, for use on wings with sweptforward trailing edges, have inversely tapered triangular plan forms with highly sweptforward hinge lines (a = -0.95). The data for the sweptforward trailing-edge case (fig. 3(b)) can be used to illustrate the effect of the $\cos \Lambda_{HL}$ term in the parameter $F_L$. The value of $F_L$ is greater for the untapered control (a = -0.40) having $\beta A_f = 8.0$ than for the inversely tapered control having a = -0.95 and $\beta A_f = 3.6$. Use of equation 1(b) indicates, however, that at Mach numbers greater than 1.29 the effect of the $\cos \Lambda_{HL}$ terms is such that the inversely tapered control has the higher value of L/H.

Figure 3(c) shows that, for wings having unswept trailing edges, the plan forms of tapered controls having maximum values of L/H are triangular in shape and have highly swept hinge lines. This figure shows identical values of L/H for normally and inversely tapered controls.

For wings having sweptback trailing edges, maximum values of L/H are shown for controls with triangular plan forms of normal taper. It will be interesting to note that ratios of L/H for the more desirable tapered and untapered controls located at the midspan position are not a great deal larger than ratios of L/H for controls having the same plan forms but located at the tip or inboard positions.

Without knowledge of the wing geometry, the maximum control span which may be used, the Mach number, and the required value of $C_{L_\delta}/A$, it is not possible to specify when tapered controls have higher values of L/H than untapered controls. When these parameters are known, however, it is simple, by use of the charts in figure 3, to determine whether untapered or tapered controls provide greater values of L/H. With the insertion in equation (4) of the maximum control span which may be used $\left(2b_f/b\right)_1$ and the required value of $\left(C_{L_\delta}/A\right)_1$, a value of $\left(2b_f/b\right)_0$ corresponding to the lift-coefficient slope $\left(\dfrac{C_{L_\delta}}{A} = 0.0001\right)$ is obtained. The value of L/H, indicated by the appropriate chart of figure 3 for an untapered control having this value of $2b_f/b$, can then be compared with values of L/H for tapered controls having this span or smaller spans and smaller aspect ratios.

### Ratios of Lift to Deflection Work

Effects of spanwise location.- In determining control locations for which ratios of lift to deflection work are maximum, the procedure is the same as in the case where hinge moment is the criterion. This is true because comparisons are made between controls of constant shape and constant $C_{L_\delta}/A$, in which cases maximum values of $F_L$ correspond to maximum values of both L/H and L/W. The conclusions regarding control locations for maximum values of L/W would therefore be the same as those regarding control locations for maximum values of L/H; that is, values of L/W for controls located in midspan positions are always equal to or higher than values for similar controls at other locations.

Effects of control plan form.- In determining control plan forms for maximum ratios of lift to deflection work, the use of the charts in figure 3 is considerably more simple than was the case in the analysis dealing with hinge moment because comparisons are made on the basis of $F_L$ rather than $\dfrac{F_L}{\cos \Lambda_{HL}}$.

All the charts for the midspan control position in figure 3 show maximum values of $F_L$, and therefore maximum ratios of lift to deflection work, for untapered controls with values of $\beta A_f = 8.0$. Since values of $L/W$ would increase with increasing values of $\beta A_f$, as discussed in the section dealing with hinge moment, it is concluded that untapered controls of maximum aspect ratio have maximum values of $L/W$. (This conclusion is similar to a result obtained in the analysis for the incompressible case of reference 2 which states that flaps should be of almost constant chord and should be as long and narrow as compatible with structural and other design considerations.) It must be remembered, however, that values of $\beta A_f$ above 8.0 would correspond to impractically high aspect ratios at relatively low supersonic Mach numbers.

It might be well to note that the advantages of untapered controls over tapered controls decrease as the wing trailing-edge sweep (either forward or back) is increased. The effects of control locations also are relatively small for the high-aspect-ratio untapered controls.

Effects of control size.- The effect of control size on the value of $L/W$ can be readily determined from equation (6). For a given amount of required lift and a given control shape, control lift-coefficient slopes are inversely proportional to control deflection, and equation (6) can be rewritten:

$$\left(\frac{L}{W}\right)_1 = \sqrt{\frac{C_{L_{\delta_1}}}{C_{L_{\delta_0}}}}\left(\frac{L}{W}\right)_0 \tag{6a}$$

It can be seen from equation (6a) that the ratio of lift to deflection work for controls of similar shape is proportional to the square root of lift-coefficient slope and also to the square root of control area since $C_{L_\delta}$ is directly proportional to control area.

## Ratios of Rolling Moment to Hinge Moment

Effect of control size.- From a comparison of the charts for the three values of $C_{l_\delta}/A$, the ratios of rolling moment to hinge moment are seen to increase with increasing values of $C_{l_\delta}/A$ and, consequently, with decreasing control size. This result is logical because the ratio of rolling moment to hinge moment is essentially a ratio of moment arms, and the ratio of moment arms increases as the size of a control of given shape decreases. This result is significant because it indicates that

control hinge moments can be appreciably reduced by using smaller controls and larger deflections.

Untapered controls.- The curves for untapered controls in figure 7 show that the rate of increase in $F_l$ with control aspect ratio increases very rapidly as the value of $C_{l_\delta}/A$ decreases; thus, high-aspect-ratio untapered controls compare favorably with the tapered controls at $\frac{C_{l_\delta}}{A} = 0.0002$. This trend appears to indicate that untapered controls will have higher ratios of rolling moment to hinge moment than tapered controls when the rolling requirements are sufficiently low $\left(\text{values of } C_{l_\delta}/A \text{ somewhat less than } 0.0002\right)$.

The aspect ratios at which maximum values of $L'/H$ occur for untapered controls with $\frac{C_{l_\delta}}{A} = 0.0002$ are beyond the range of the calculations. For $\frac{C_{l_\delta}}{A} = 0.0004$ and $0.0006$, however, untapered controls having maximum values of $L'/H$ are shown to have spans roughly between 60 and 80 percent of the semispan of wings having unswept and sweptback trailing edges, regardless of the value of $\beta A_f$. Controls having spans between 60 and 80 percent of the wing semispan are also shown to have maximum values of $L'/H$ on wings having sweptforward trailing edges when values of $\beta A_f$ less than 4 are neglected. It thus appears that span is the important parameter for defining the plan forms of untapered controls having maximum values of $L'/H$, except possibly for wings having sweptforward trailing edges.

The indication that span is the important parameter can be somewhat substantiated by means of plane geometry if it is assumed that the controls are uniformly loaded and that no loads are carried on the wing (thus, it is possible to work with simple area moments). For any arbitrary rolling moment, the ratio of rolling moment to hinge moment for a control located at the wing tip can be shown to increase with control span until it reaches a maximum value when the control span is two-thirds of the wing semispan. It seems logical that this type of analysis would be applicable except for low-aspect-ratio controls or low Mach numbers; for these cases, the conical-flow regions are very large and cannot be neglected.

Tapered controls and sweptforward trailing edges.- Figures 7(a) and 7(b) show that, on wings having sweptforward trailing edges, inversely tapered controls having triangular plan forms and highly sweptforward

hinge lines (a = -0.95) in practically all cases, provide maximum values of  L'/H.

Tapered controls and unswept trailing edges.- The data in the chart for  $\dfrac{C_{l_\delta}}{A}$ = 0.0002  in figure 7(c) show considerably greater values of  $F_l$ for high-aspect-ratio untapered controls than for the tapered controls (a = ±0.6).  At Mach numbers greater than 1.91, however, the normally and inversely tapered triangular controls both have higher ratios of rolling moment to hinge moment than does the untapered control of 0.65b/2 span.  (See eq. (1b).)  On the basis of figure 7(c), triangular plan forms having absolute values of  a  greater than 0.6 would be expected to have higher values of  L'/H  than untapered controls of 0.65b/2 span at Mach numbers considerably less than 1.91.  It is therefore concluded that, for  $\dfrac{C_{l_\delta}}{A}$ = 0.0002  and at moderate and high Mach numbers, maximum values of  L'/H  are obtained by use of triangular plan forms and highly swept hinge lines.  Although normally tapered triangular plan forms have somewhat higher values of  L'/H  than do inversely tapered triangular plan forms, it is probable that, because of structural considerations of the supporting wings, the inversely tapered controls are more practical when the hinge lines are highly swept.  The data in the charts for  $\dfrac{C_{l_\delta}}{A}$ = 0.0004 and 0.0006, although showing very little difference in values of  $F_l$  for controls having hinge lines swept forward and swept back (a = 0.6 and -0.6, respectively), indicate maximum values of  L'/H  for inversely tapered controls having triangular plan forms and highly sweptforward hinge lines.

Tapered controls and sweptback trailing edges.- For wings having sweptback trailing edges, figures 7(d) and 7(e), in general, show maximum values of  L'/H  for normally tapered controls (a = 0.8).  Since the effects of normal taper on the area distribution of controls is such that reduced values of  L'/H  would be ordinarily expected, it seems probable that the advantage of normally tapered controls results from their larger regions of conical flow.  The importance of such regions can be surmised from the charts for  $\dfrac{C_{l_\delta}}{A}$ = 0.0004 and 0.0006, where optimum values of  $\beta A_f$ are near the minimum values shown on the curves.  (The minimum values shown on these curves, as throughout figure 7, are near the values at which the Mach cones from the inboard-control parting lines intersect the control tips, corresponding to comparatively large regions of conical flow.)  The charts for  $\dfrac{C_{l_\delta}}{A}$ = 0.0002  show that, from minimum to maximum values of

$\beta A_f$, values of $L'/H$ for normally tapered controls first increase to maximum values and then decrease. Because the areas of the conical-flow regions decrease consistently with increasing control aspect ratio, there is evidently some parameter more important than the areas of the conical-flow regions which causes values of $L'/H$ to increase as values of $\beta A_f$ are increased. This parameter is probably control-area distribution because it has been shown that, for untapered controls, increased values of $L'/H$ can be obtained by increasing the aspect ratios and spans of controls having spans of less than about two-thirds the wing semispan. Figures 7(d) and 7(e), therefore, appear to indicate that plan forms of tapered controls on wings having sweptback trailing edges, for which maximum values of $L'/H$ exist, are dependent on the interrelated parameters, control-area distribution and conical-flow area, and cannot be generally specified.

As a matter of practical interest, differences between the hinge-line ($a = 0.8$) and trailing-edge sweep angles' corresponding to the Mach number range between 1.3 and 2.5 are roughly between $15^\circ$ and $19^\circ$ in figures 7(d) and between $7^\circ$ and $8^\circ$ in figure 7(e). These differential angles are sizable, and it is probable that, on wings having relatively small differences between the leading- and trailing-edge sweep angles, smaller differences will be of more practical interest. If somewhat smaller differences, corresponding to values of $a$ which are nearer to values of $\dfrac{\tan \Lambda_{TE}}{\beta}$, are considered, figures 7(d) and 7(e) indicate that advantages of tapered controls over untapered controls are, in general, relatively small and at values of $\dfrac{C_{l_\delta}}{A} = 0.0002$ are probably nonexistent.

<div align="center">Ratios of Rolling Moment to Deflection Work</div>

In using the charts in figure 7 to determine the plan forms of controls having maximum ratios of rolling moment to deflection work at a given value of $C_{l_\delta}/A$, the various curves are compared directly $\left(\text{on the basis of } F_l\right)$. For controls having different values of $C_{l_\delta}/A$, however, maximum values of $F_l$ do not necessarily correspond to maximum values of $L'/W$ (since different control deflections are required to produce a fixed rolling moment), and comparisons of such controls must therefore be made on the basis of values of $F_l/\delta_l$ rather than $F_l$.

<u>Effects of control plan form</u>.- With the exception of figure 7(a), all the data for $\dfrac{C_{l_\delta}}{A} = 0.0002$ in figure 7 show maximum values of $L'/W$

for high-aspect-ratio untapered controls. In figure 7(a), higher values of $L'/W$ are shown for inversely tapered triangular controls with highly swept hinge lines. The data for the three values of $C_{l_\delta}/A$ appear to indicate, however, that high-aspect-ratio untapered control plan forms have maximum values of $L'/W$ at values of $C_{l_\delta}/A$ somewhat less than 0.0002. It might therefore be concluded that, for sufficiently small controls, maximum values of $L'/W$ are in all cases obtained by use of high-aspect-ratio untapered plan forms. The spans of untapered controls for maximum values of $L'/W$, as discussed for maximum values of $L'/H$, would probably be of the order of two-thirds the wing semispan.

For controls having values of $C_{l_\delta}/A$ of 0.0004 and 0.0006, the data of figures 7(a) and 7(b) for wings with sweptforward trailing edges show maximum values of $L'/W$ for inversely tapered controls having triangular plan forms. For wings having unswept trailing edges, the effects of plan form on values of $L'/W$ are shown in figure 7(c) to be relatively small. Untapered controls with spans of about two-thirds the wing semispan, however, are shown to have values of $L'/W$ which are equal to or greater than those for other control plan forms. Figures 7(d) and 7(e) show maximum values of $L'/W$ on wings having sweptback trailing edges for normally tapered controls with values of $\beta A_f$ between 3 and 5. As mentioned in the analysis dealing with hinge moments, the differences between hinge-line and trailing-edge sweep angles for the normally tapered controls of figures 7(d) and 7(e) are for many applications impractically high; and, for controls having values of a near values of $\dfrac{\tan \Lambda_{TE}}{\beta}$, which are probably of more practical interest, the data indicate that values of $L'/W$ would be little, if any, higher than those for untapered controls with spans of about two-thirds the wing semispan.

For purposes of comparison, it is of interest at this point to note the results obtained in the analysis of plan form for the incompressible case (ref. 2). These results are: The shape of ailerons for minimum deflection work is of maximum width near the wing tip and has a slight convex curvature as it tapers to zero chord at the center of the wing (somewhat similar to the sweptforward trailing-edge case of the present analysis). Partial-span ailerons should be sections of these shapes and should include the regions of maximum chord. The ailerons should be as long and narrow as compatible with structural and other design considerations.

Effects of control size.- When the data for the different values of $C_{l_\delta}/A$ in figure 7 were used to determine effects of control size on the value of $L'/W$, it was necessary to use a fixed value of $C_l/A$ as a basis for comparison because of the $\delta$ term in $F_l$. A value of $\dfrac{C_l}{A} = 0.0006$

was arbitrarily chosen for which values of $\delta_1 = 3$, $1\frac{1}{2}$, and 1 are
required, respectively, for controls with values of $C_{l_\delta}/A$ of 0.0002,
0.0004, and 0.0006. The comparisons were then made by dividing values
of $F_l$ from the various charts by corresponding values of $\delta$. Results
of the comparisons for control plan forms previously discussed as having
higher values of $L'/W$ are presented in the following table:

| Figure | $\dfrac{\tan \Lambda_{TE}}{\beta}$ | $a$ | $\beta A_f$ | $\dfrac{2b_f}{b}$ | $\dfrac{\beta}{2}\dfrac{L'}{W}$ at $\dfrac{C_{l_\delta}}{A}$ of - | | |
|---|---|---|---|---|---|---|---|
| | | | | | 0.0002 | 0.0004 | 0.0006 |
| 7(a) | -0.60 | -0.95 | 5.7 | Varied | 11.3 | 13.2 | 13.6 |
| 7(b) | -.40 | -.95 | 3.6 | Varied | 8.8 | 10.7 | 11.3 |
| 7(c) | 0 | 0 | Varied | 0.65 | 8.0 | 8.0 | 8.2 |
| 7(d) | .40 | .40 | Varied | .60 | 9.1 | 8.8 | 8.9 |
| 7(e) | .60 | .60 | Varied | .65 | ---- | 10.9 | 10.6 |

The data in the above table show that, for wings having sweptforward
trailing edges, there are appreciable increases in values of $L'/W$ with
increasing size of inversely tapered controls. For wings having unswept
and sweptback trailing edges, little effect of the size of untapered
controls on values of $L'/W$ is shown. It would thus appear, especially
when the relieving effect of hinge moments due to rolling are considered,
that larger controls would, in most cases, have somewhat higher ratios of
rolling moment to deflection work than would smaller controls.

## VARIATIONS WITH MACH NUMBER OF THE CHARACTERISTICS OF

### EXAMPLE LATERAL CONTROL CONFIGURATIONS

#### Conditions

Specifications.- It is very difficult from the analysis charts of
figure 7 to visualize the characteristics of lateral control surfaces on
aircraft operating over a speed range. In order to illustrate the vari-
ation with Mach number of the characteristics of some of the control plan
forms which have been shown to be desirable, some example calculations
have been made. The specifications used for the calculations are as
follows: Wings having spans of 38 feet are to be equipped with aileron
controls capable of producing rolling rates of 3.0 radians per second
while operating at Mach numbers up to 2.25 at altitudes of 40,000 feet.
Wings are to have trailing-edge sweep angles of $-20^\circ$, $0^\circ$, and $35^\circ$ with

other plan-form variables unspecified. Combined deflections of ailerons on opposite wing panels are not to exceed $30^\circ$.

Determining values of $C_{l_\delta}/A$ required.- In figure 8 are presented, as a function of Mach number, wing-tip helix angles $pb/2V$ corresponding to the above specified conditions of wing span, rate of roll, and altitude. In order to determine the rolling moments required to produce these wing-tip helix angles, the wing damping-moment coefficients must be known. Figure 9 presents the theoretical damping-moment coefficients for a broad range of wing plan form and Mach number obtained by use of charts presented in references 4 and 8. In order to calculate the required rolling moments without fixing the wing plan forms, it is necessary to make the simplifying assumption that damping-moment coefficients do not change with Mach number; figure 10 has been prepared for the purpose of examining such an assumption and shows that damping coefficients of highly swept wings are relatively independent of Mach number and that damping coefficients of high-aspect-ratio wings are influenced to a greater extent by Mach number than the damping coefficients of low-aspect-ratio wings. Figure 10 shows that the results obtained in the present paper by assuming fixed damping-moment coefficients are directly applicable to moderate- and low-aspect-ratio wings having highly swept leading edges.

Rolling-moment coefficients corresponding to the wing-tip helix angle in figure 8 and to fixed damping-moment coefficients, which were considered to be representative on the basis of figure 9, are presented in figure 11. In order to determine rolling-moment-coefficient slopes corresponding to the values of $C_l/A$ presented in figure 11, it is only necessary to divide the values of $C_l/A$ by $30^\circ$. In order to provide a more practical example, however, some consideration should be given to the effects of wing thickness, nonlinearities of control effectiveness with control deflection, and wing flexibility, which are known to result in actual values of control effectiveness which are considerably less than the theoretical values. The illustrative example of reference 1 shows that, when an approximate thickness correction was applied, the effectiveness of a control on a 5-percent-thick wing was reduced to about 80 percent of that predicted by theory. On the basis of this example, and by making an arbitrary allowance for nonlinearities, it was assumed that the effectiveness of controls on rigid wings would be 60 percent of that predicted by theory. A further assumption, quite arbitrarily made, was that the effectiveness of controls on flexible wings would be 60 percent of that for controls on rigid wings and, consequently, 36 percent of the theoretical values. Estimated values of $C_{l_\delta}/A$ necessary to produce the required values of $C_l/A$ (fig. 11) were obtained by use of the preceding assumptions and are presented in figure 12.

Control plan forms.- Rolling-moment-coefficient slopes and ratios of rolling moment to hinge moment were calculated through a Mach number range for several control plan forms on wings having trailing-edge sweep angles of -20°, 0°, and 35° and having damping-moment coefficients $-C_{l_p}/A$ of 0.08 (chosen as a mean value from fig. 9). For each configuration, calculations were made for untapered control plan forms having spans of approximately 50, 65, and 80 percent of the wing semispan and for tapered control plan forms having various hinge-line sweep angles. Calculations of L'/H were also made for representative control plan forms on wings having unswept trailing edges and having damping coefficients $-C_{l_p}/A$ of 0.03, 0.08, and 0.14. Results of the latter calculations were also used to illustrate the effects of control plan form and size on ratios of rolling moment to deflection work.

## Results of L'/H Computations

As can be seen by the charts for $C_{l_\delta}/A$ in figure 13, all the control plan forms, for which results are presented, provide rolling moments equal to or greater than those which were estimated to be necessary to meet the required specifications (fig. 12).

Untapered controls.- Maximum values of L'/H for untapered controls are shown in figure 13 for controls having spans of about 65 percent of the wing semispan except at low Mach numbers on wings having trailing-edge sweep angles of -20°. In this case, the control having a span of 50 percent of the wing semispan provides slightly higher values of L'/H. These results illustrate the previously discussed conclusion that untapered-control plan forms for maximum values of L'/H have spans of about two-thirds the wing semispan except in cases of low control aspect ratios or low Mach numbers. It might be well to point out that the advantage of the 0.65 semispan flaps over the 0.50 and 0.80 semispan flaps is small. If compared with flaps having spans of less than one-half the semispan or greater than four-fifths the semispan, however, the advantage of the 0.65 semispan flaps would be expected to be greater.

Figure 13 shows that the values of L'/H for untapered controls increase as the controls are swept either forward or back. This result is probably due to the fact that the center of loading of the control, which remains near the same chordwise location regardless of sweep, is nearer the hinge line when the control is swept than when it is unswept. The above speculation can be somewhat substantiated because it can be shown that, by dividing the values of L'/H shown for the unswept case by appropriate $\cos \Lambda_{TE}$ terms, values of L'/H for the swept controls

can be roughly approximated. Thus the values of $L'/H$ for the sweptback trailing edge are greater than those for the sweptforward trailing edge, principally because of the greater sweep rather than the direction of sweep.

Tapered controls.- For wings having trailing-edge sweep angles of -20° and 0° (fig. 13), consistent increases in the values of $L'/H$ with $\Lambda_{HL}$ are shown at any given Mach number. At the maximum design Mach number of 2.25, inversely tapered controls having hinge lines swept forward 60° provide values of $L'/H$ roughly 50 percent greater than those shown for untapered controls with $\frac{2b_f}{b} = 0.65$. At lower Mach numbers, a still greater advantage is shown for the inversely tapered controls. Simply stated, this means that the inversely tapered controls require, at most, only about two-thirds the hinge moment required by untapered controls to produce the required rolling moments.

For wings having sweptback trailing edges, the tapered control plan forms were chosen by fixing the hinge-line sweep angles at 40° and 45° (believed to be practical values for $\Lambda_{TE} = 35°$) and using figure 7 to estimate the more desirable aspect ratios. Results presented in figure 13 show that ratios of $L'/H$ for the tapered controls are somewhat greater than for untapered controls at the lower Mach numbers but slightly less at the higher Mach numbers. Figure 13 indicates that greater values of $L'/H$ could probably be obtained at substantially high Mach numbers with tapered controls if a considerably greater amount of hinge-line sweep were used. Aside from being structurally impractical, it would appear from figure 13 that such controls would have an extremely high rate of decrease in the values of $L'/H$ with Mach number.

Effect of varying damping coefficients.- Because figure 7 indicates that control plan forms for maximum values of $L'/H$ vary somewhat with the amount of rolling moment required, rolling moment and ratio of rolling moment to hinge moment have been calculated for example controls on wings having unswept trailing edges and having damping-moment coefficients $-C_{l_p}/A$ of 0.03, 0.08, and 0.14. For each damping-moment coefficient, calculations were made for untapered controls having spans of approximately 65 percent of the wing semispan and for inversely tapered triangular controls having hinge lines swept forward 60°. Results of these calculations are presented in figure 14.

In figure 14 data are presented for inversely tapered triangular controls at Mach numbers for which the hinge lines are swept behind the Mach lines (indicated by dashed lines). These data were obtained by use of equations presented in reference 3 and are of particular interest because they show that this type of control, which has been shown to have

high values of $L'/H$ at the higher Mach numbers, produces satisfactory rolling moments and increasing values of $L'/H$ as the Mach number is decreased. The ratios of rolling moment to hinge moment increase very rapidly as control size (with constant span for untapered controls) is decreased (fig. 14). This condition illustrates, as did figure 7, the advantage of using small controls and maximum practical deflections. Figure 14 also shows that the advantage of the inversely tapered control over the untapered control decreases steadily with decreasing rolling-moment requirements until, for rolling moments corresponding to

$$\frac{-C_{l_p}}{A} = 0.03,$$ the untapered control has a higher value of $L'/H$ at a Mach

number of 2.25. It might be pointed out, however, that the untapered control in this case has the very high, and perhaps somewhat impractical, aspect ratio of 16.5.

## Results of $L'/W$ Computations

In order to illustrate some effects of control plan form and size on the ratios of rolling moment to deflection work, sample calculations were made for the controls shown in figure 14. It was assumed for the calculations that the rolling requirements of the controls were the same as in the previous examples and that the wing damping-moment coefficient $-C_{l_p}/A$ was 0.03. The upper chart in figure 15 presents the theoretical rolling-moment requirements for an assumed practical control effectiveness which is 36 percent of the theoretical values. (It should be mentioned that the deflections necessary to produce the required rolling moments vary considerably with Mach number, as well as with control size, because of varying values of $C_{l_\delta}/A$.) The lower chart in

figure 15 presents the ratios of rolling moment to deflection work required to produce this rolling moment.

The data in figure 15 for the untapered controls illustrate the previously discussed conclusion regarding the effect of control size; that is, for controls having spans of about two-thirds the wing semispan, ratios of rolling moment to deflection work are not appreciably influenced by control size. In consideration of hinge moments due to rolling, however, it is probable that maximum values of $L'/W$ will in practice be obtained by use of the larger controls.

The data in figure 15 for the inversely tapered controls show sizable increases in values of $L'/W$ with control size. Similar results for this type of control on wings having sweptforward trailing edges, as previously mentioned, were indicated by the analysis charts.

SUMMARY OF RESULTS

Theoretical analyses have been made to determine the plan forms of
unbalanced trailing-edge flap-type controls having minimum hinge moments
due to deflection and requiring minimum work to overcome the hinge
moments due to deflection at supersonic speeds. Ratios of lift and
rolling moment to hinge moment and ratios of lift and rolling moment to
deflection work at fixed values of lift and rolling effectiveness were
used as bases for the analyses. Hinge moments due to angle of attack
and damping in pitch and rolling, which are dependent on wing plan form
and to varying degrees on complete aircraft configuration, have not been
included in the present analyses and will have to be taken into account
in applying results of these analyses to any particular wing.

Results of the analyses are summarized in a table and are as follows:
For longitudinal controls, maximum ratios of lift to hinge moment $L/H$
are obtained with untapered controls of maximum aspect ratio. In prac-
tice it is, in many cases, not possible to obtain sufficient lift with
high-aspect-ratio controls; when moderate and low-aspect-ratio controls
must be used, controls with triangular plan forms and highly swept hinge
lines have higher values of $L/H$ than untapered controls. The plan forms
of triangular controls having maximum values of $L/H$ are inversely
tapered for wings with sweptforward trailing edges and are normally
tapered for wings with sweptback trailing edges. On wings with unswept
trailing edges, direction of hinge-line sweep is of little importance.
For control plan forms having maximum values of $L/H$, control location
is of little importance.

Maximum ratios of lift to deflection work are shown for untapered
controls of high aspect ratio. In contrast with the results regarding
hinge moment, untapered controls require less deflection work than
tapered controls when the lift requirements are such that controls of
moderate, and in some cases low, aspect ratio must be used.

For any given control shape, the analyses for lateral controls illus-
trate the importance of using small controls with high deflections for
obtaining maximum ratios of rolling moment to hinge moment $L'/H$. Control
plan form, although secondary to control size, is also shown to be very
important. For wings having sweptforward or unswept trailing edges,
inversely tapered controls having triangular plan forms and highly swept-
forward hinge lines are shown to have maximum values of $L'/H$. For wings
having sweptback trailing edges, control plan forms for maximum values of
$L'/H$ are dependent on the particular requirements and cannot be generally
specified. Results indicate, however, that, for such wings, little can be
gained in practice by tapering the control. The spans of untapered con-
trols having maximum values of $L'/H$ are shown in most cases to be of
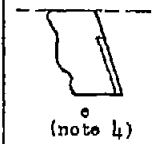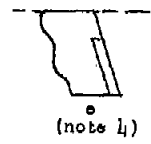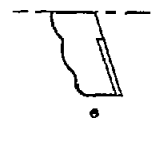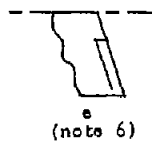the order of two-thirds the wing semispan.
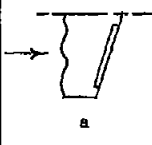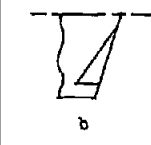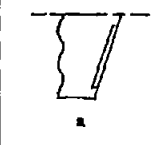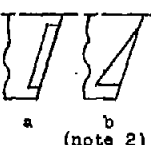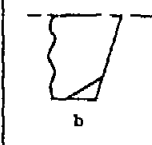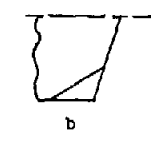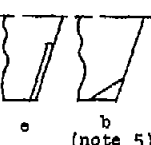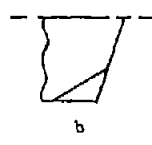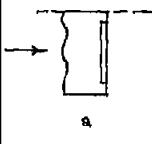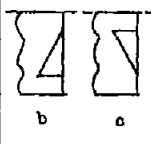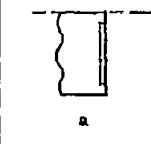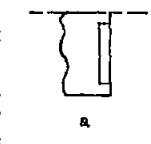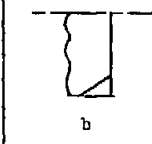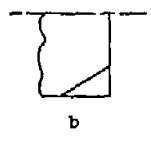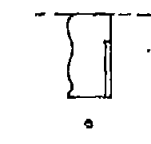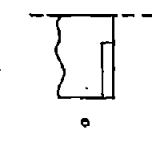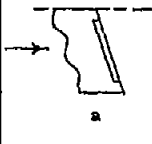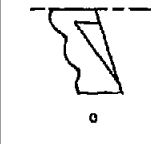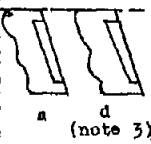
When the rolling requirements are low enough to permit the use of very small controls, maximum ratios of rolling moment to deflection work $L'/W$ are in all cases indicated for untapered controls of high aspect ratio. When more conventional control sizes are necessary, maximum values of $L'/W$ on wings with sweptforward trailing edges are shown for inversely tapered controls with triangular plan forms. On wings with unswept and sweptback trailing edges, effects of hinge-line sweep are not of especial importance; when the more practical configurations are considered, untapered controls with spans of about two-thirds the wing semispan are indicated to have near maximum values of $L'/W$. Effects of control size on values of $L'/W$ for these untapered controls are shown to be negligible. For the inversely tapered controls on wings with sweptforward trailing edges, however, values of $L'/W$ are shown to increase appreciably with control size. Since hinge moments due to rolling increase with control size, it would thus appear that large controls would in most cases require less deflection work than smaller controls.


Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., Nov. 19, 1952.

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TABLE I.- PLAN FORMS OF CONTROLS GENERALLY SHOWN TO HAVE MAXIMUM RATIOS OF LIFT AND ROLLING MOMENT TO HINGE MOMENT

DUE TO DEFLECTION AND MAXIMUM RATIOS OF LIFT AND ROLLING MOMENT TO WORK REQUIRED TO OVERCOME THE HINGE

MOMENTS DUE TO DEFLECTION.  (FOR MORE COMPLETE DESCRIPTIONS SEE SUMMARY OF RESULTS.)

| Wing trailing-edge sweep condition | Plan forms for maximum L/H | | Plan forms for maximum L/W | | Plan forms for maximum L'/H | | Plan forms for maximum L'/W | |
|---|---|---|---|---|---|---|---|---|
| | Small control | Large control | Small control | Large control | Small control | Large control | Small control | Large control |
| Sweptforward | a | b | a | a    b (note 2) | b | b | e    b (note 5) | b |
| Unswept | a | b    c | a | a | b | b | e | e |
| Sweptback (note 1) | a | c | a | a    d (note 3) | e (note 4) | e (note 4) | e | e (note 6) |

a.  High-aspect-ratio untapered control plan forms.

b.  Inversely tapered triangular control plan form , highly sweptforward hinge line.

c.  Normally tapered triangular control plan form , highly sweptback hinge line.

d.  Normally tapered control plan form.

e.  High-aspect-ratio untapered control plan form, $\frac{2b_f}{b} \approx \frac{2}{3}$.

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Note:  1.  Tip controls shown in some cases have less rolling effectiveness than similar controls located further inboard.

2.  L/W maximum for plan form  b  when wing trailing edge is highly sweptforward and low-aspect-ratio control is required.

3.  L/W maximum for plan form  d  when wing trailing edge is highly sweptback and moderate or low-aspect-ratio control is required.

4.  Plan form  d  will for some conditions of Mach number and trailing-edge sweep have higher values of  L'/H.

5.  L/W maximum for plan form  b  when wing trailing edge is highly sweptforward.

6.  Plan form  d  will for some conditions of Mach number and trailing-edge sweep have higher values of  L'/W.
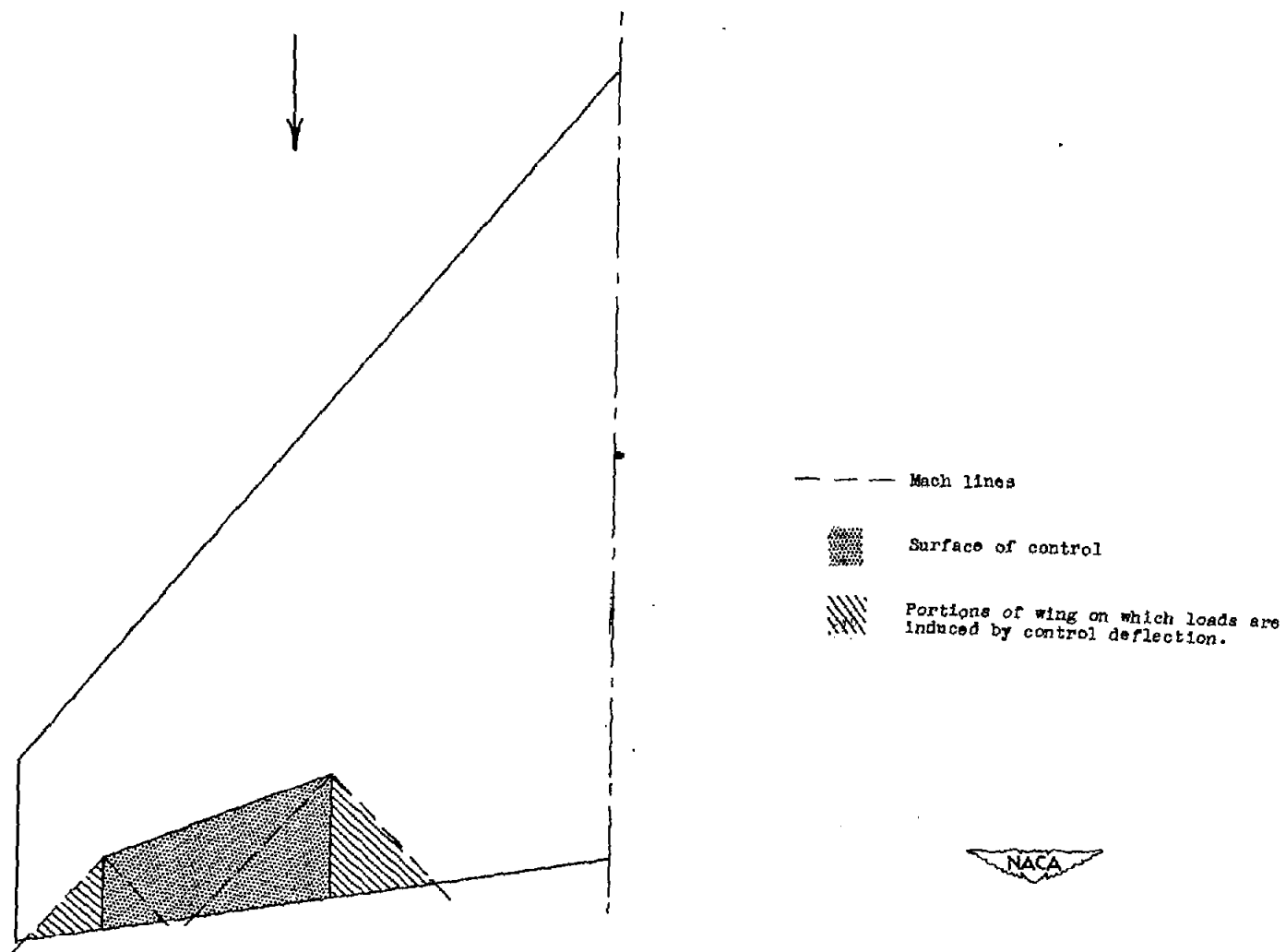
Figure 1.- Illustration of regions in which loading is influenced by the deflection of trailing-edge controls at supersonic speeds.

Tip position

Mid-span position

——— ——— ——— Mach lines

—+— —+— —+— Limiting Mach line locations

——— Assumed reflection planes

Inboard position
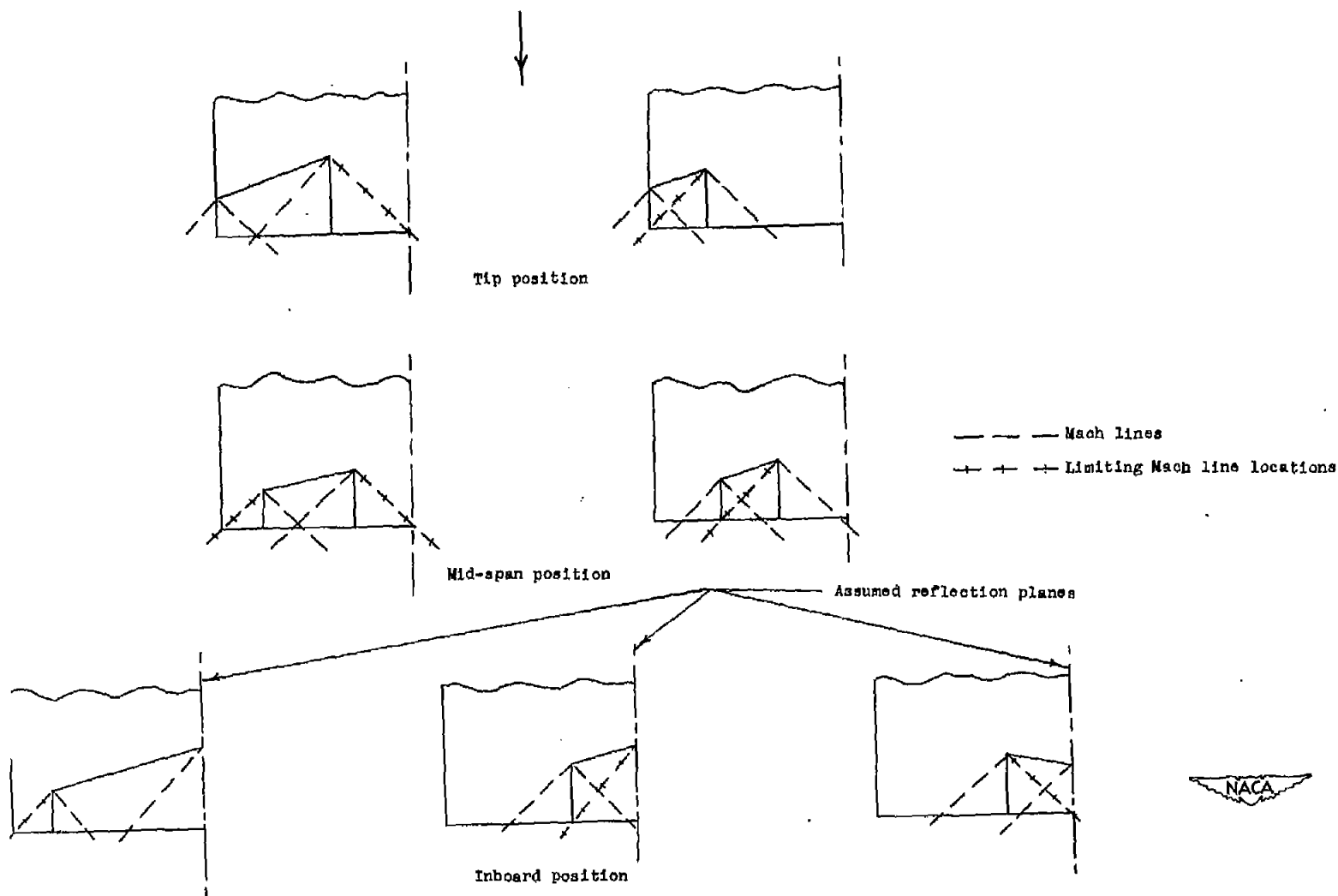
Figure 2.- Illustration of limiting Mach line locations for various
positions of longitudinal control surfaces.
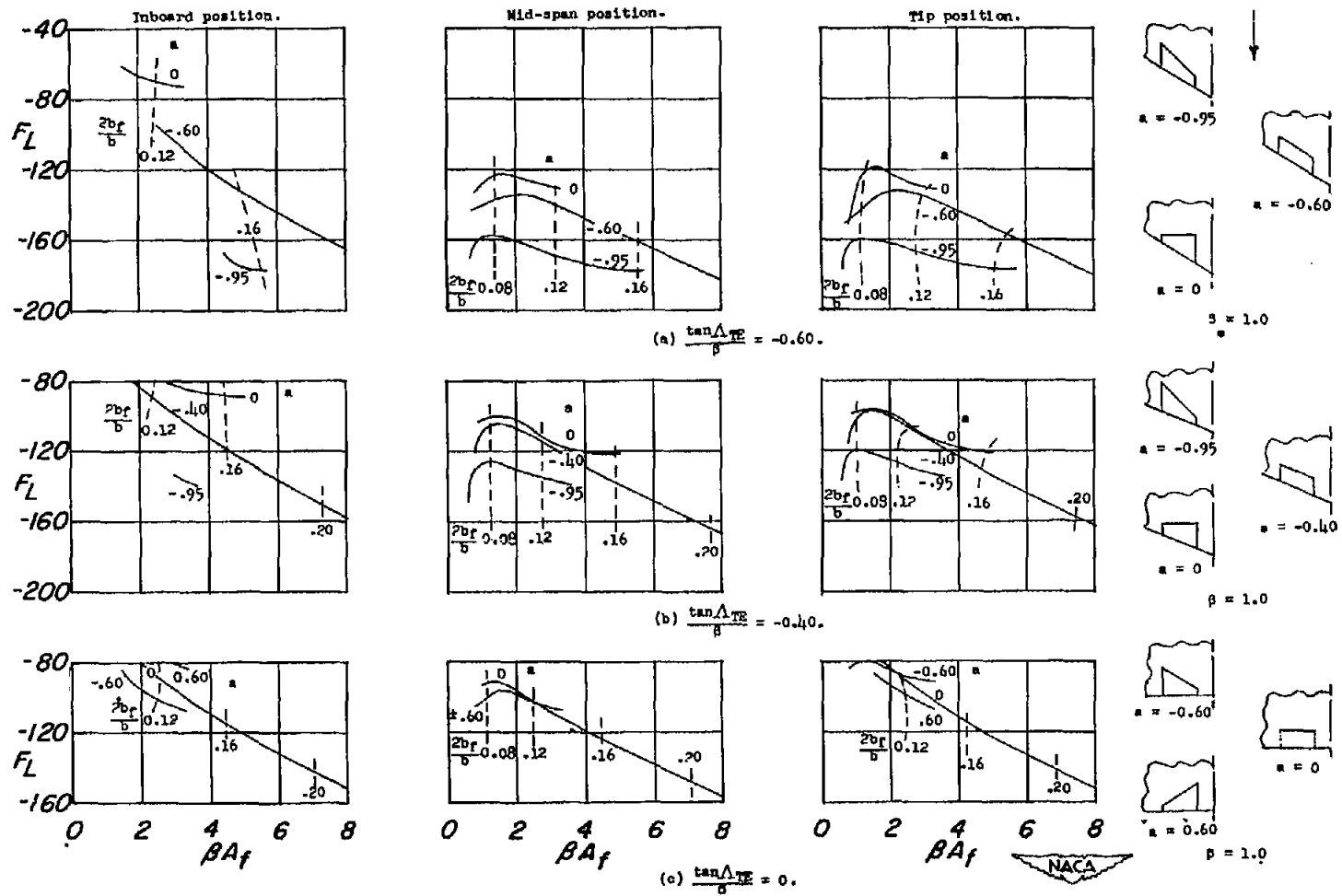
Figure 3.- Variations with control plan form and location of the parameter $F_L$ when $\dfrac{C_{l_\delta}}{A} = 0.0001.$ $\left(F_L = \beta b \cos \Lambda_{HL} \dfrac{L}{H} = \dfrac{\delta_1}{57.3} \dfrac{\beta b}{2} \dfrac{L}{W}\right)$
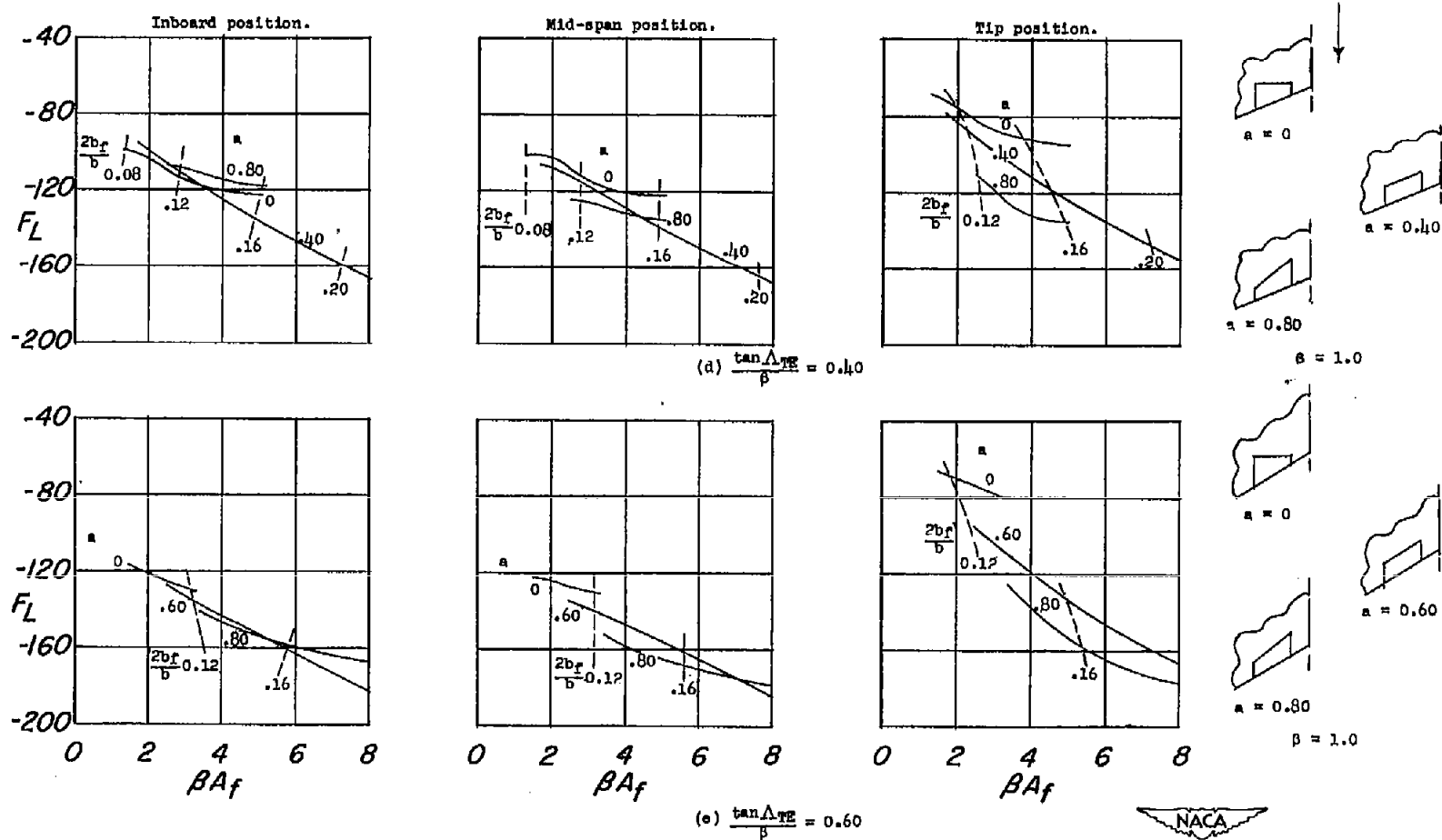
Figure 3.- Concluded.
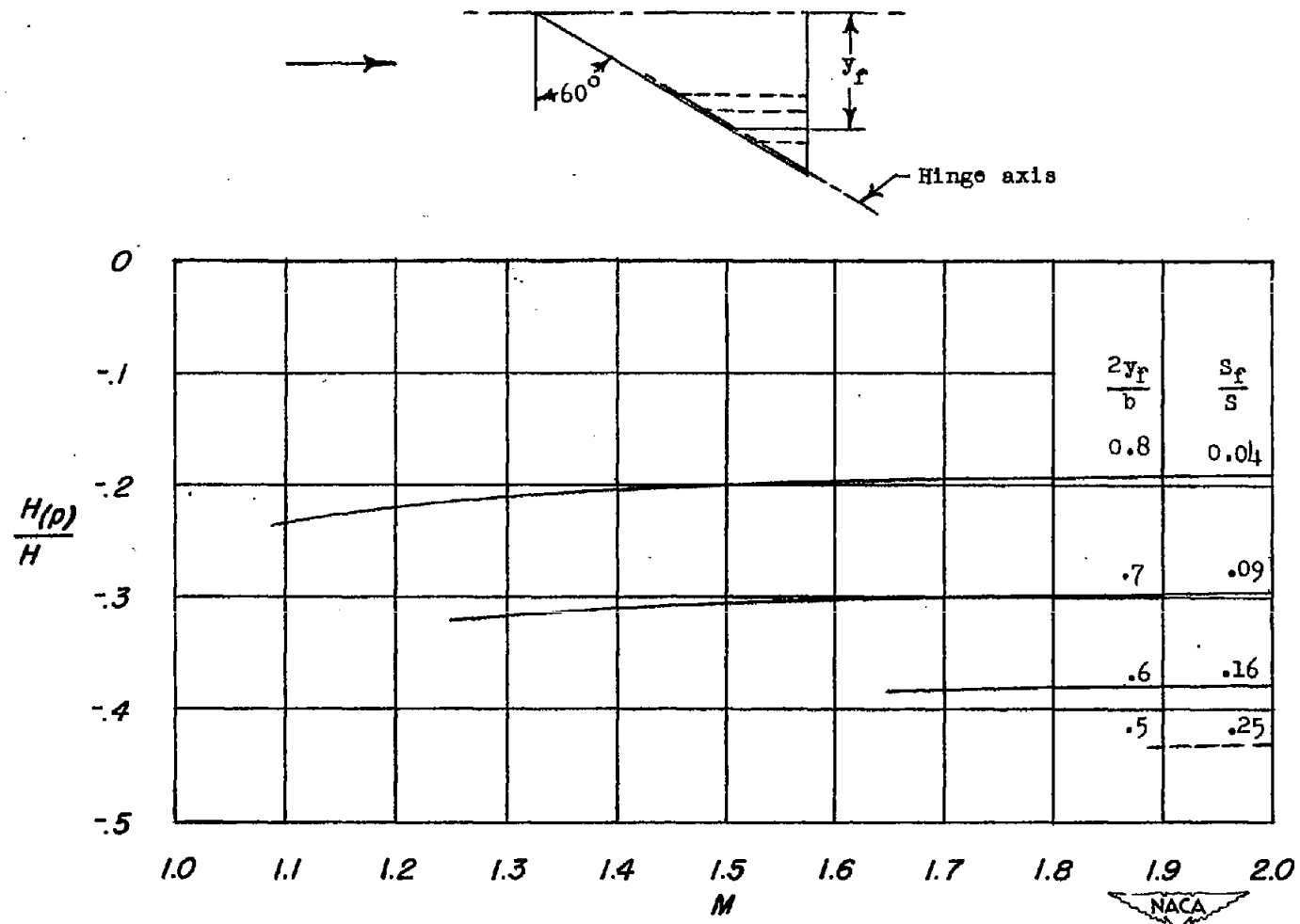
Figure 4.- Ratio of hinge moment due to steady rolling to hinge moment
due to deflection for tip controls of various sizes on a 60° delta
wing (rolling moment produced by control assumed equal to wing
damping moment).
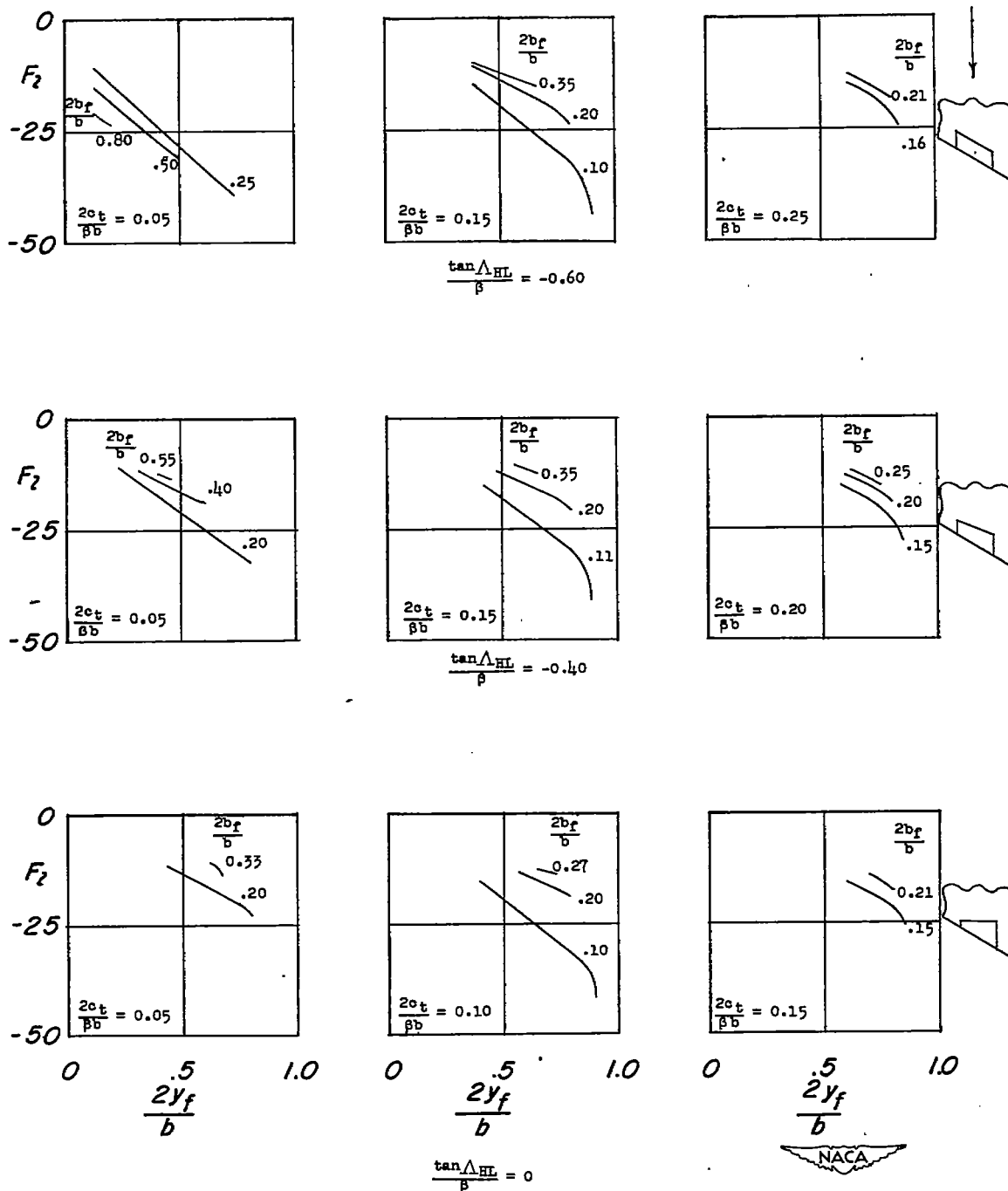
(a) $\frac{\tan\Lambda_{TE}}{\beta} = -0.60$

Figure 5.- Variations with spanwise location of the parameter $F_l$ for various control plan forms. $\left(F_l = \beta \cos \Lambda_{HL} \dfrac{L'}{H} = \dfrac{\delta_l}{57.3} \dfrac{\beta}{2} \dfrac{L'}{W}\right)$

$$\frac{\tan\Lambda_{HL}}{\beta} = -0.40$$

$$\frac{\tan\Lambda_{HL}}{\beta} = 0$$

$$\frac{2y_f}{b}$$

$$\frac{\tan\Lambda_{HL}}{\beta} = 0.40$$

(b) $\frac{\tan\Lambda_{TE}}{\beta} = -0.40$

Figure 5.- Continued.

(c) $\dfrac{\tan \Lambda_{TE}}{\beta} = 0$

Figure 5.- Continued.

(d) $\dfrac{\tan \Lambda_{TE}}{\beta} = 0.40$

Figure 5.- Continued.

$$\frac{\tan \Lambda_{HL}}{\beta} = 0.60$$

$$\frac{\tan \Lambda_{HL}}{\beta} = 0.80$$
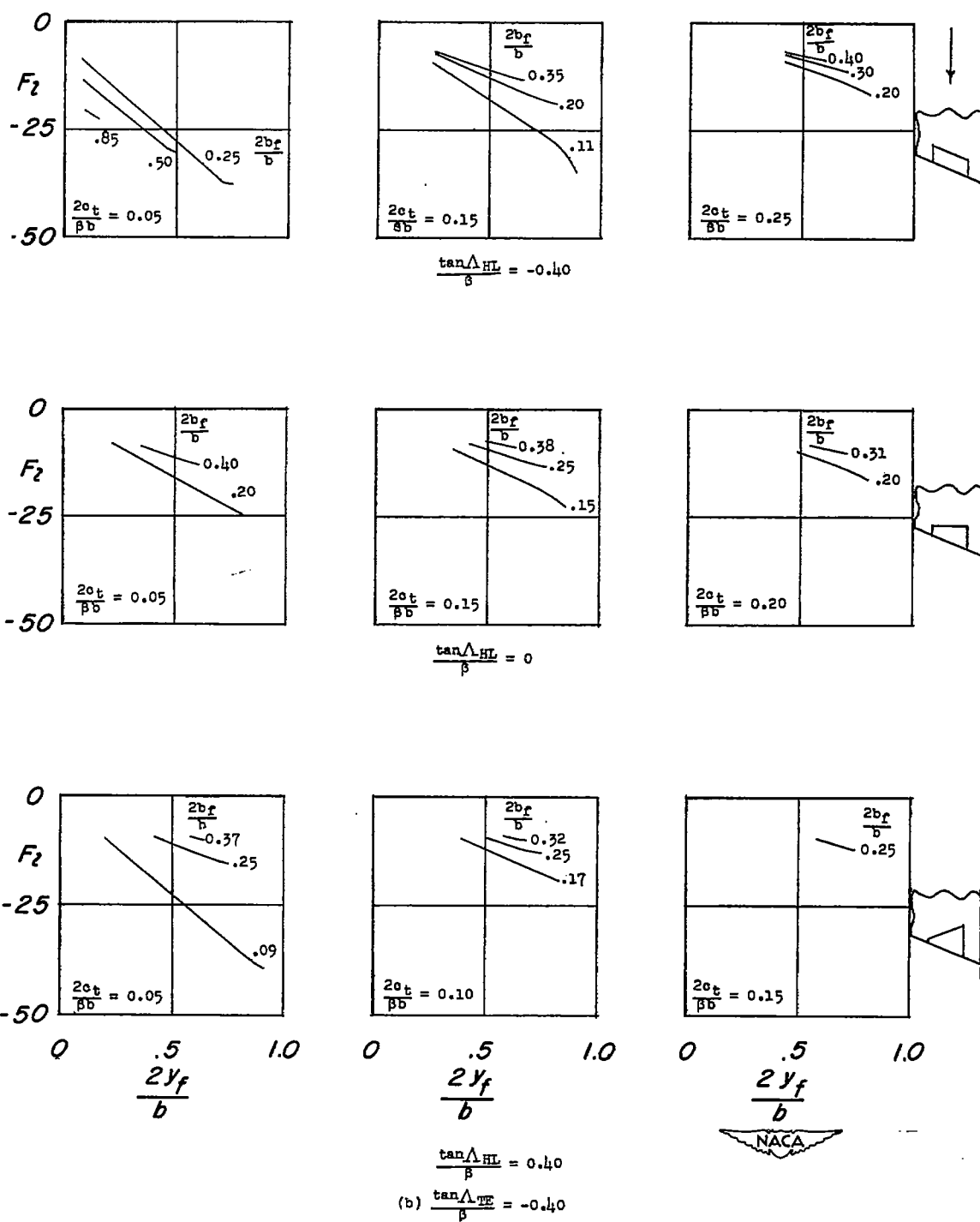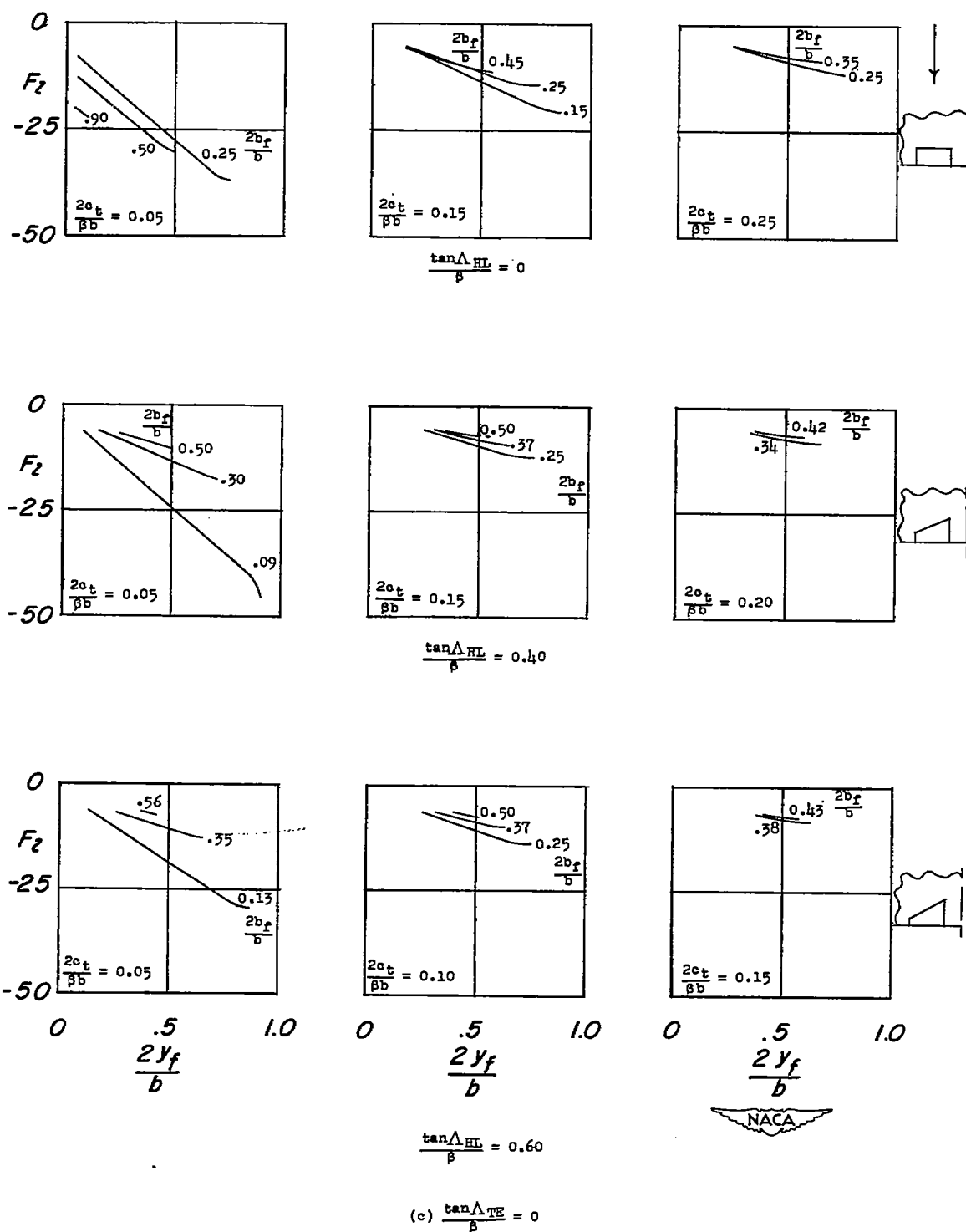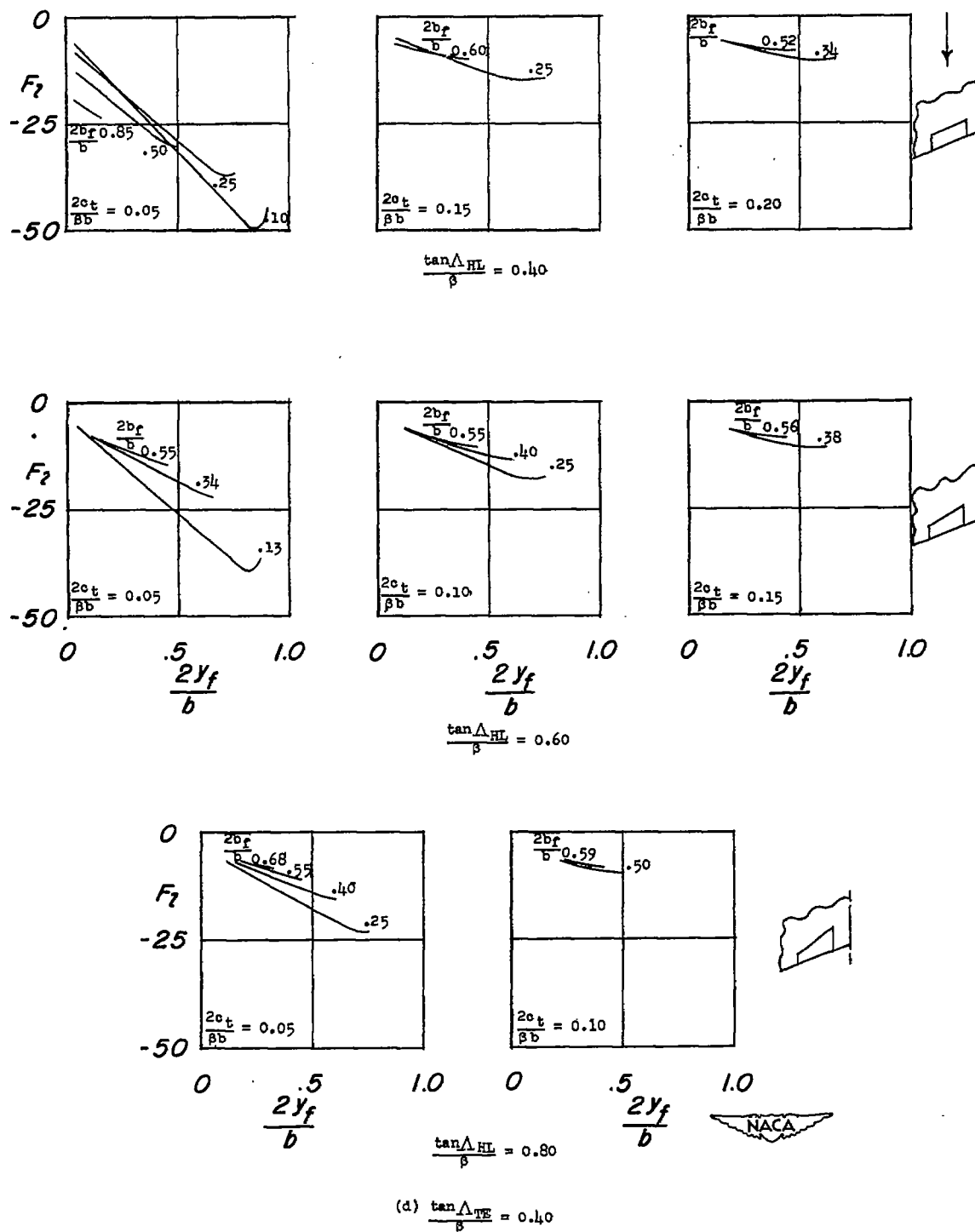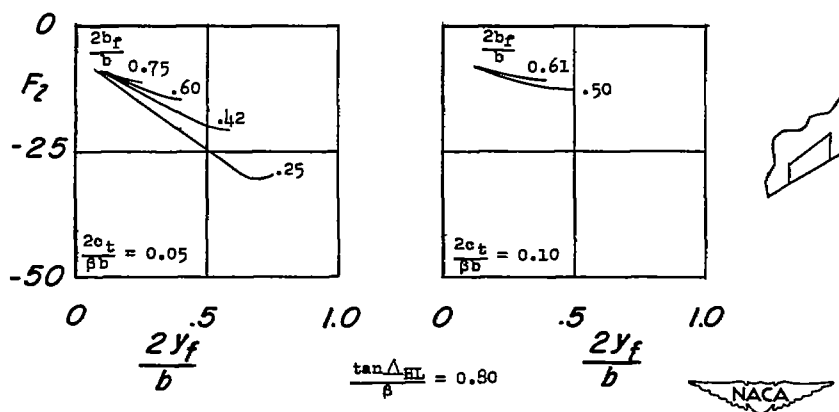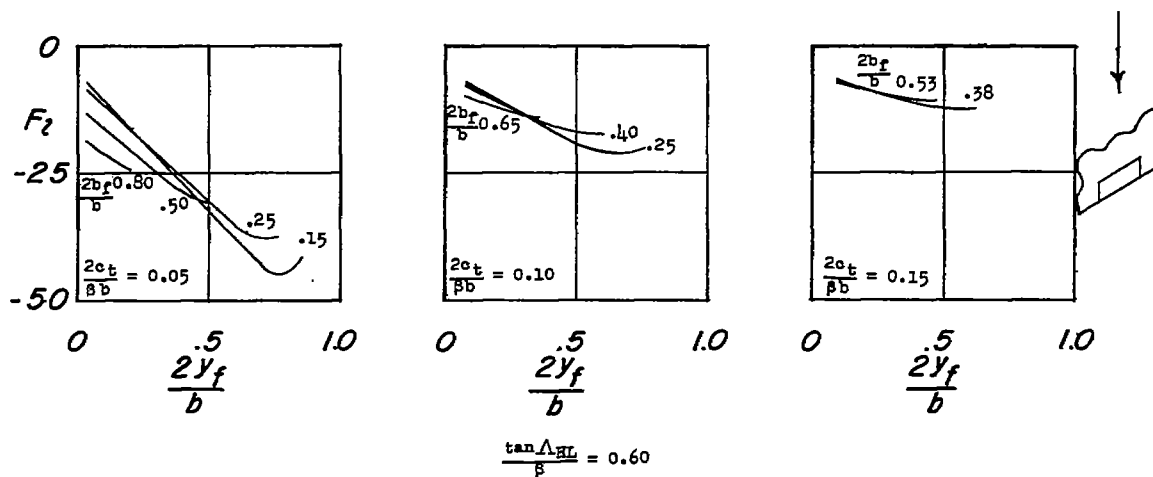
(e) $\frac{\tan \Lambda_{TE}}{\beta} = 0.60$
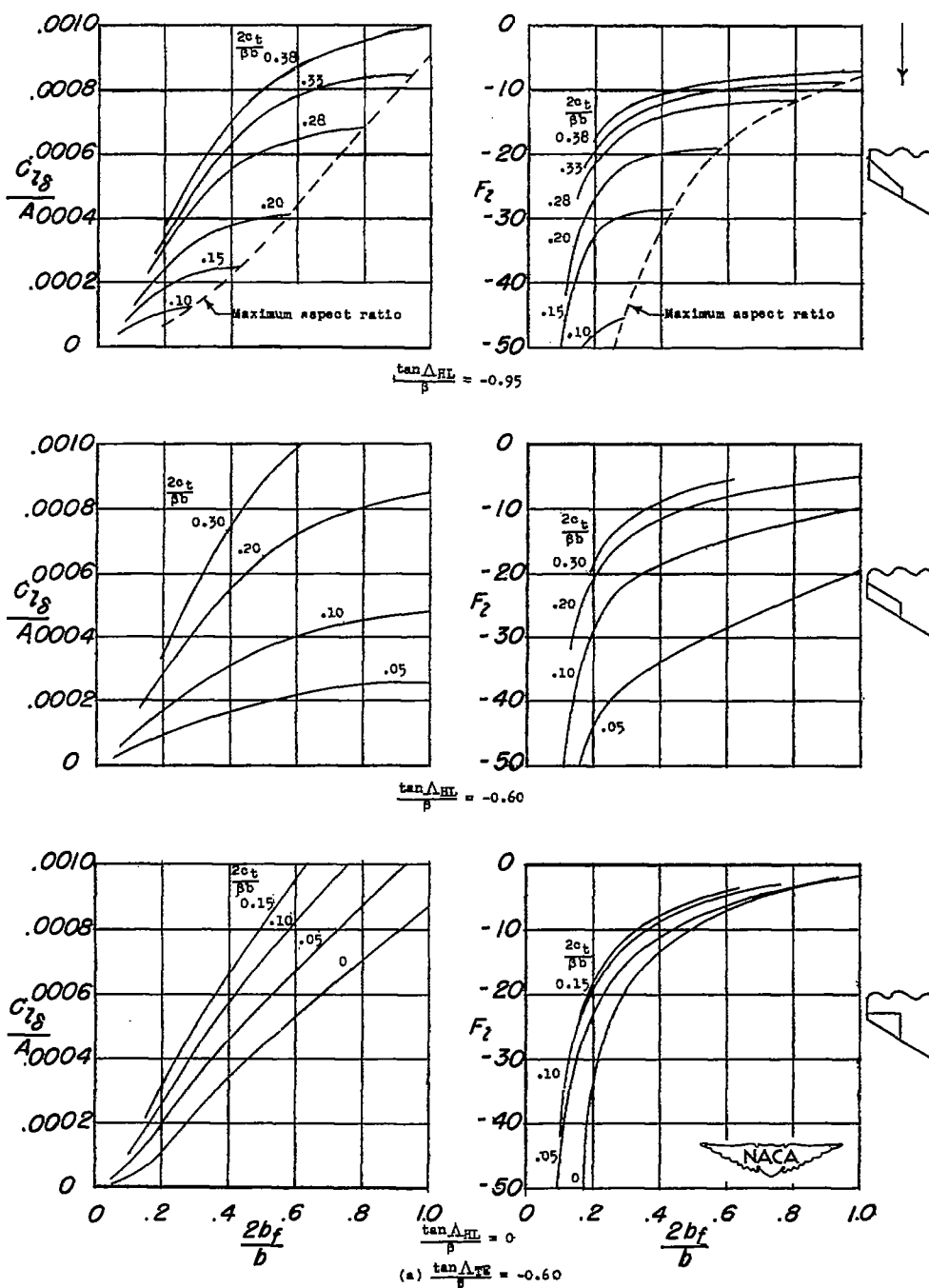
Figure 5.- Concluded.

Figure 6.- Variations with shape and size of the values of $\dfrac{C_{l_\delta}}{A}$ and $F_l$

for controls located at wing tips. $\left(F_l = \beta \cos \Lambda_{HL} \dfrac{L'}{H} = \dfrac{\delta_1}{57.3} \dfrac{\beta}{2} \dfrac{L'}{W}\right)$

$$\frac{\tan \Lambda_{HL}}{\beta} = -0.95$$

$$\frac{\tan \Lambda_{HL}}{\beta} -0.40$$

$$\frac{\tan \Lambda_{HL}}{\beta} = 0$$

(b) $\frac{\tan \Lambda_{TE}}{\beta} = -0.40$

Figure 6.- Continued.

$$\frac{\tan\Lambda_{HL}}{\beta} = -0.60$$

$$\frac{\tan\Lambda_{HL}}{\beta} = 0$$

$$\frac{\tan\Lambda_{HL}}{\beta} = 0.60$$

$$(c)\ \frac{\tan\Lambda_{TE}}{\beta} = 0$$

Figure 6.- Continued.

(d) $\dfrac{\tan \Lambda_{TE}}{\beta} = 0.40$
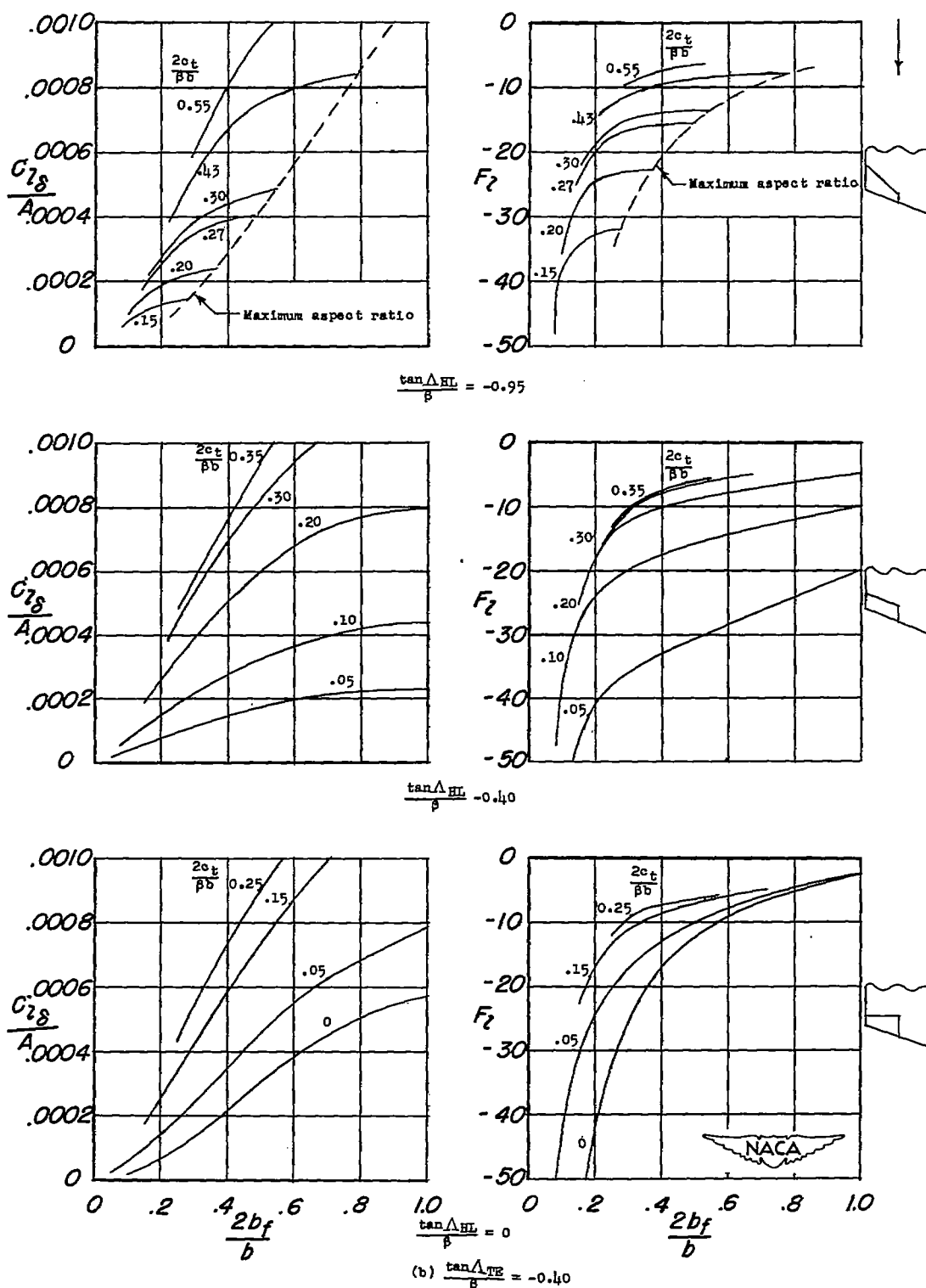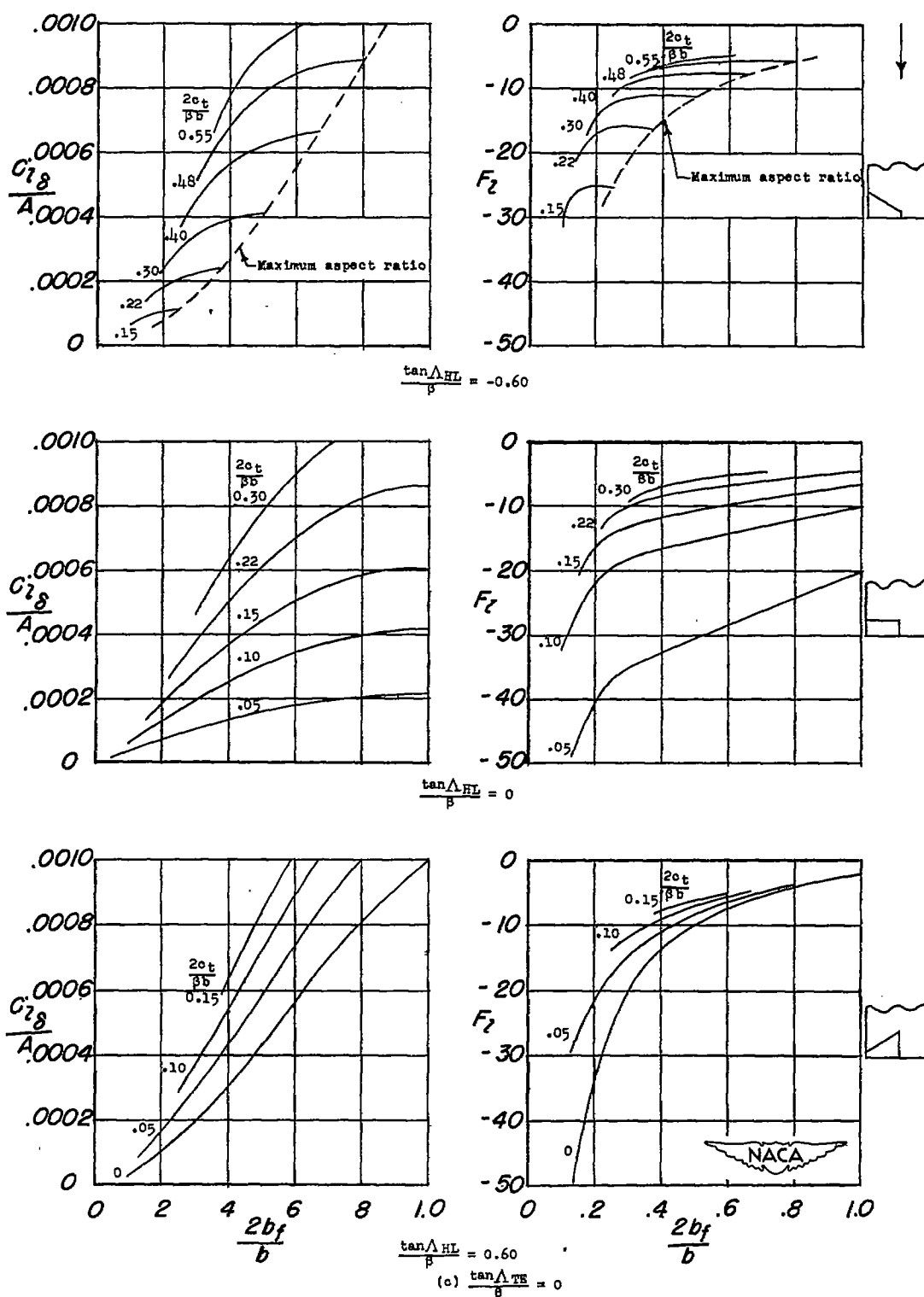
Figure 6.- Continued.
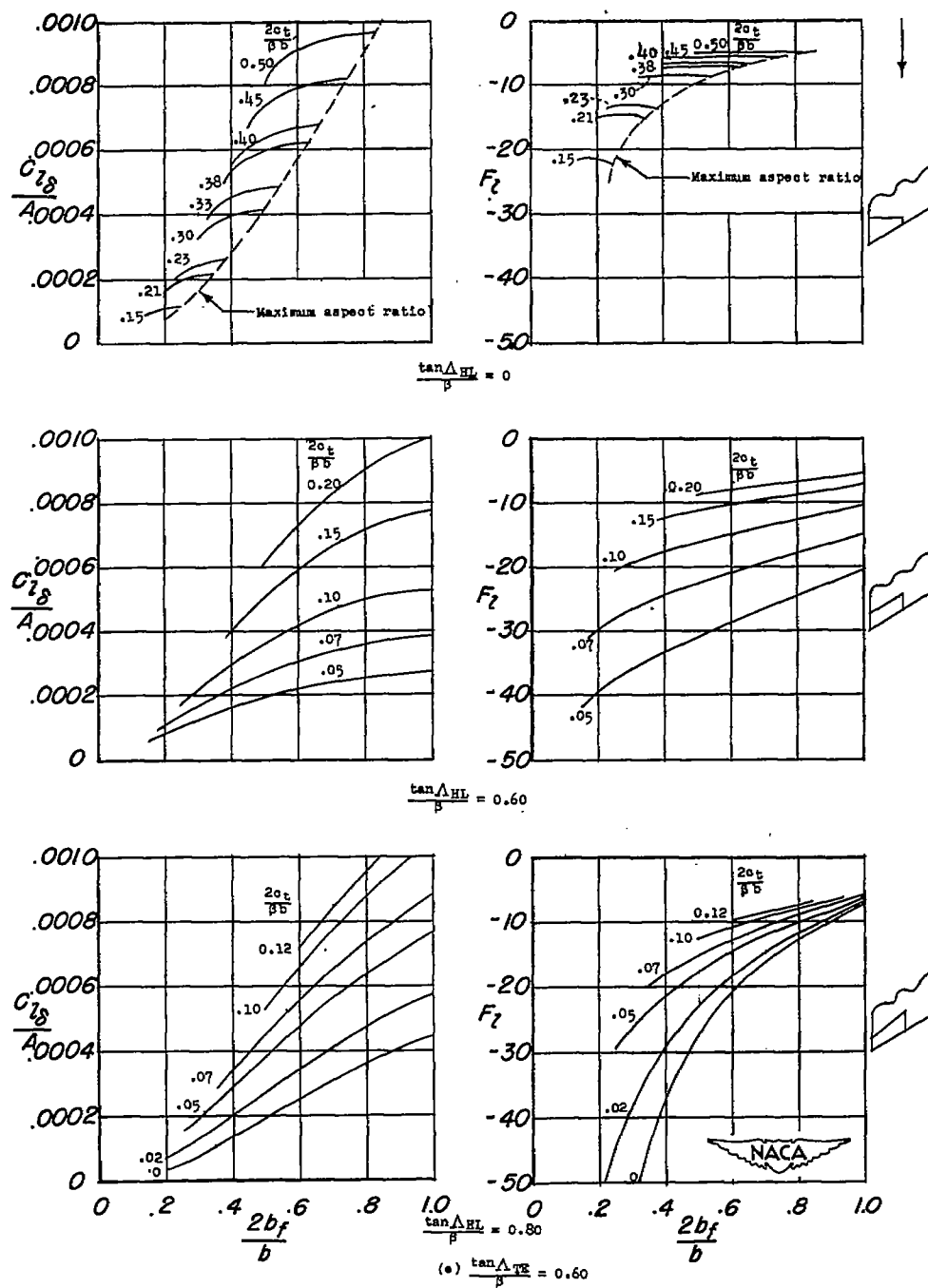
Figure 6.- Concluded.
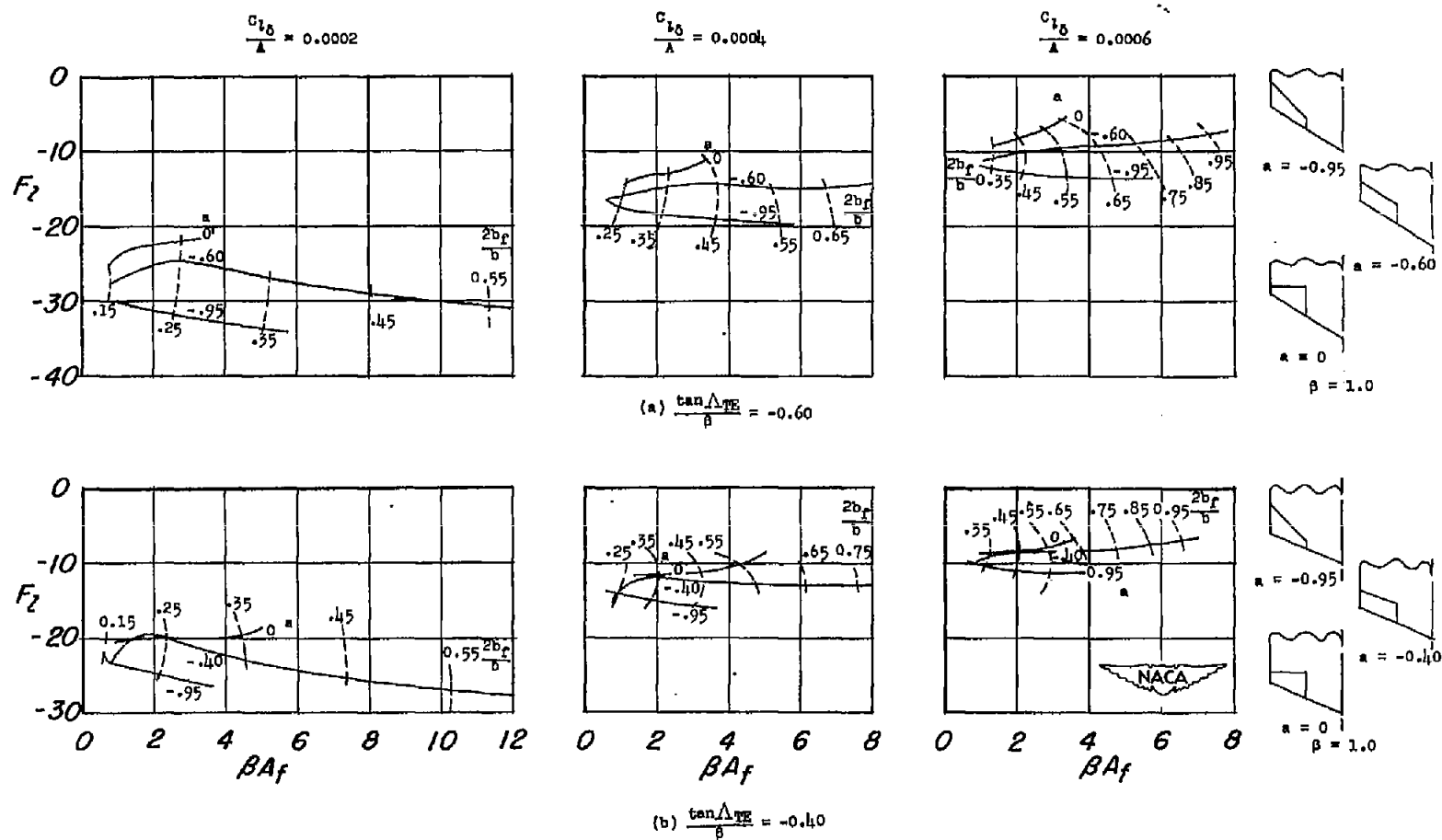
Figure 7.- Variations with plan form of the values of $F_l$ for controls located at wing tips and having values of $\dfrac{C_{l\delta}}{A} = 0.0002,\ 0.0004,$ and $0.0006.\ \left(F_l = \beta \cos \Lambda_{HL}\ \dfrac{L'}{H} = \dfrac{\delta_l}{57.3}\ \dfrac{\beta}{2}\ \dfrac{L'}{W}\right)$
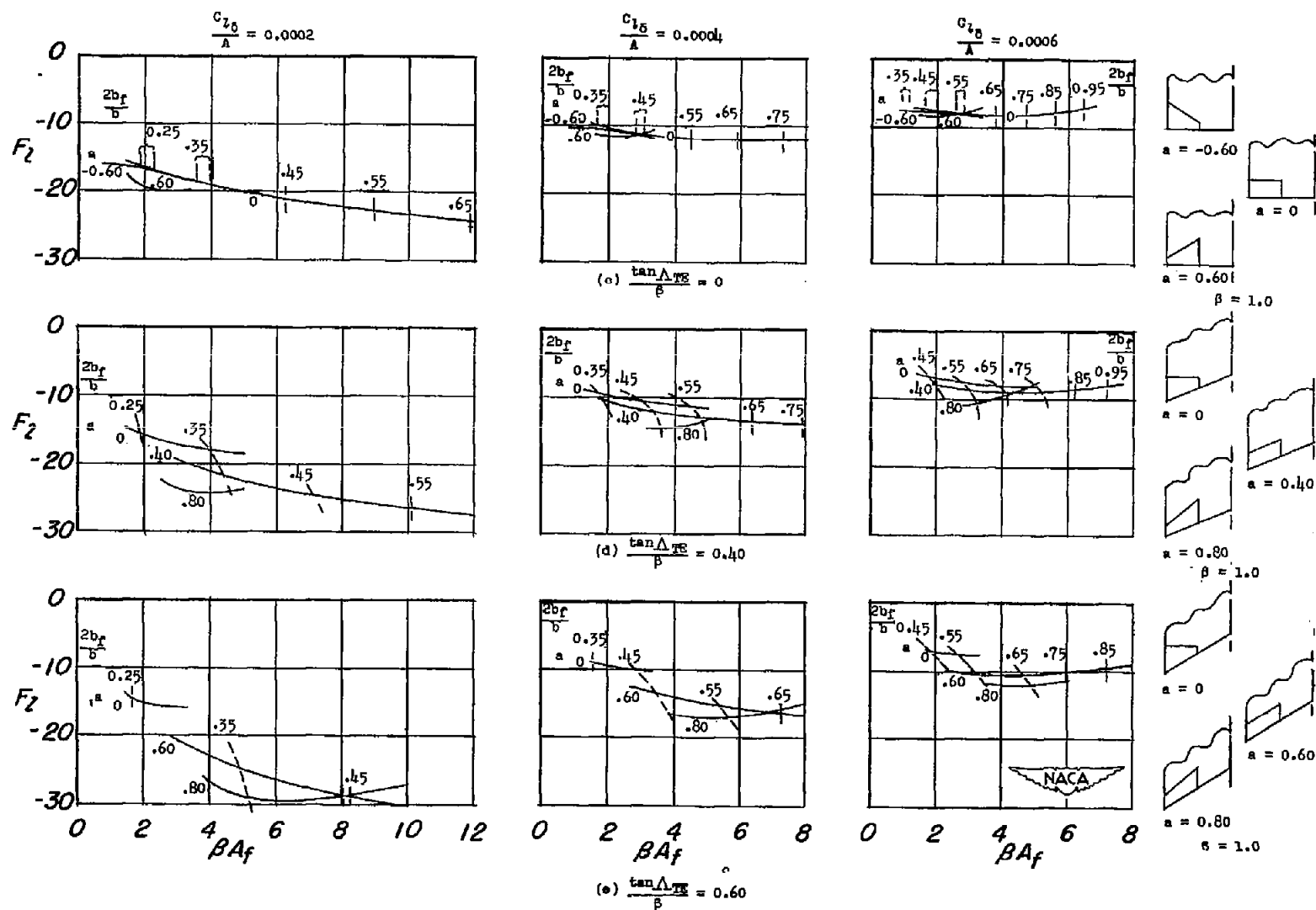
Figure 7.- Concluded.
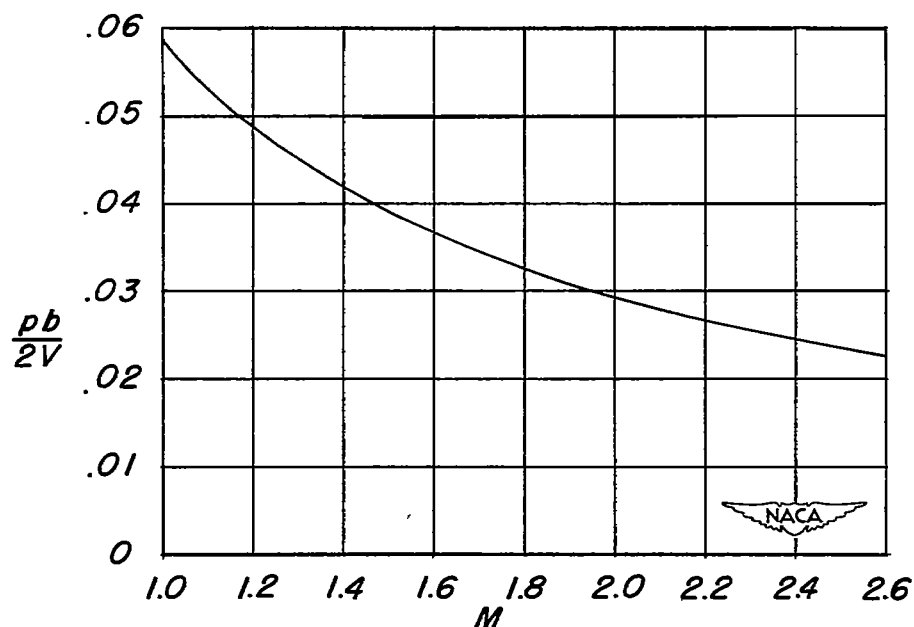
Figure 8.- Wing-tip helix angles corresponding to rolling rates of
3 radians per second for wings having spans of 38 feet and operating
at an altitude of 40,000 feet.
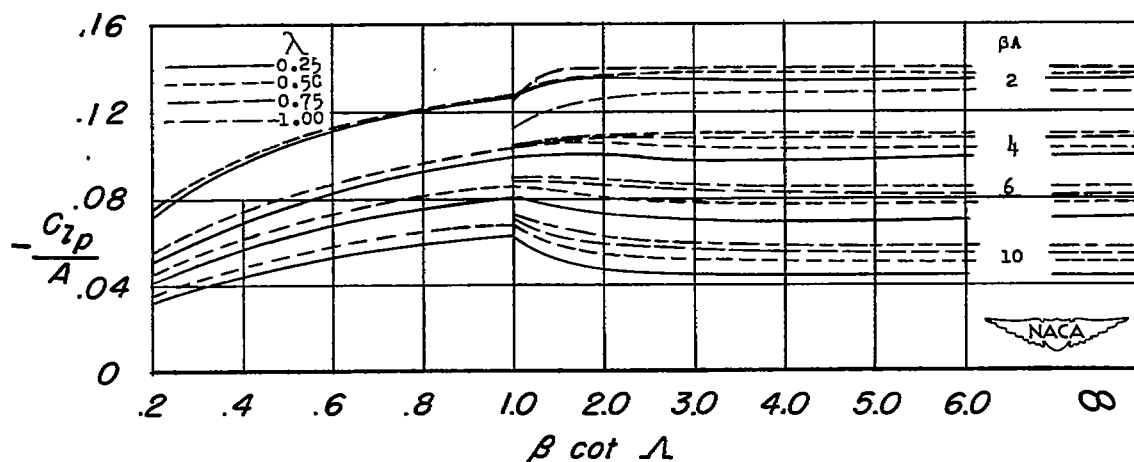


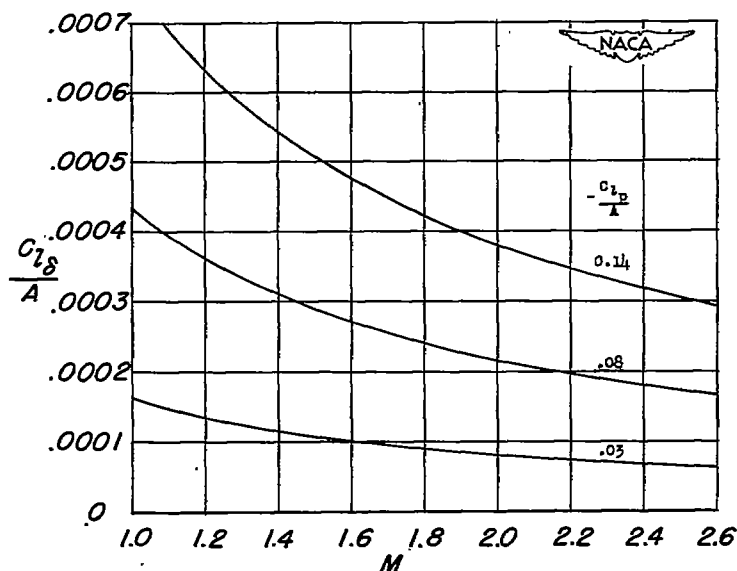Figure 9.- Theoretical damping-moment coefficients for a range of wing
plan form and Mach number.
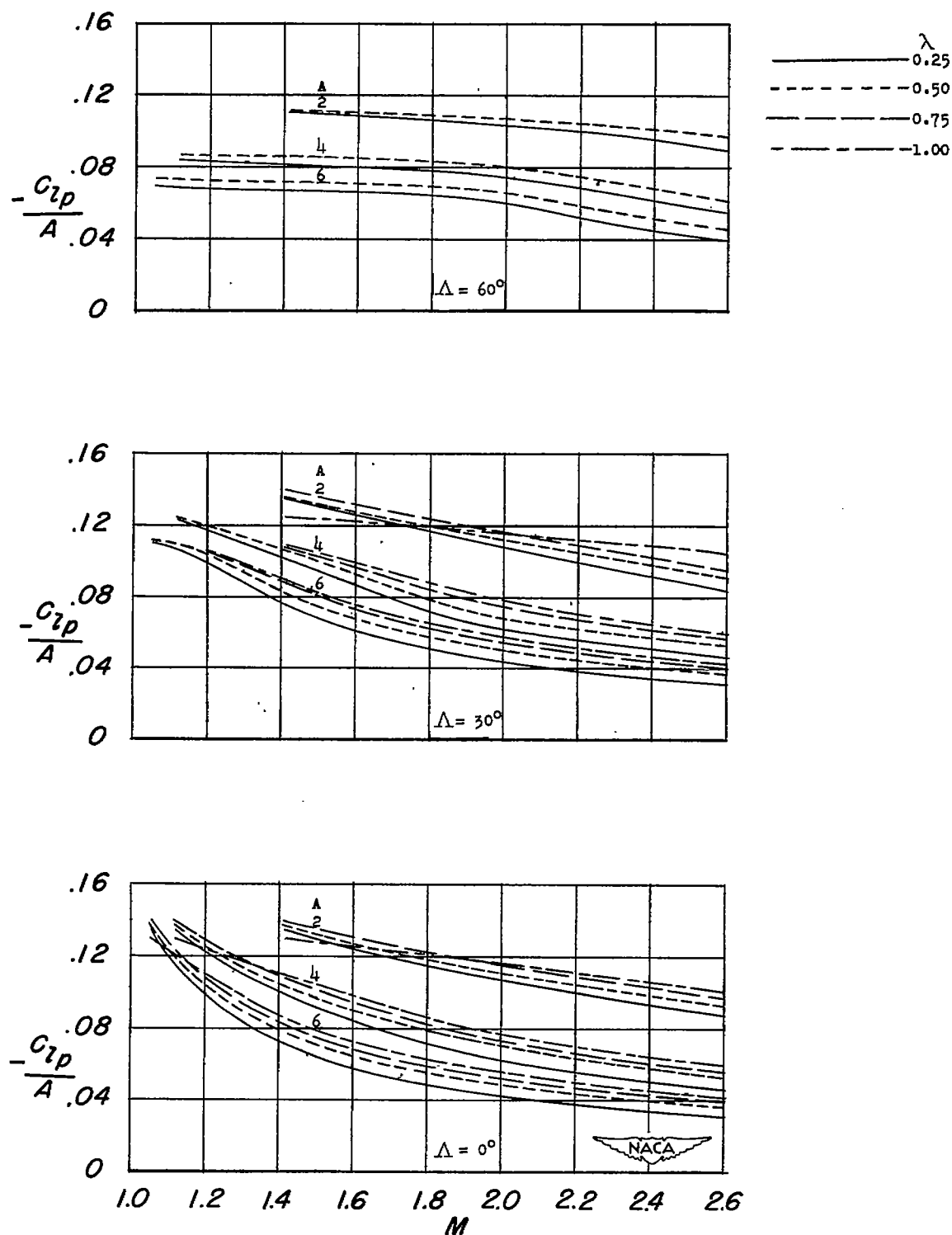
Figure 10.- Some illustrative variations of wing damping-moment
coefficients with Mach number.
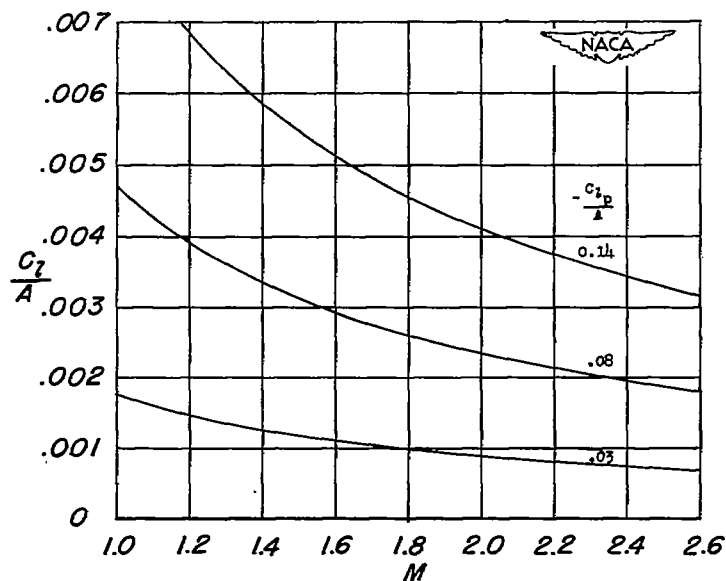
Figure 11.- Rolling-moment coefficients required to produce rolling rates
of 3 radians per second for wings having spans of 38 feet and operating
at an altitude of 40,000 feet.



Figure 12.- Estimated theoretical rolling-moment-coefficient slopes
corresponding to rolling-moment coefficients presented in figure 11,
practical effectiveness of control $\dfrac{C_{l_\delta}}{A}$ being considered as 36 per-
cent of theoretical effectiveness.
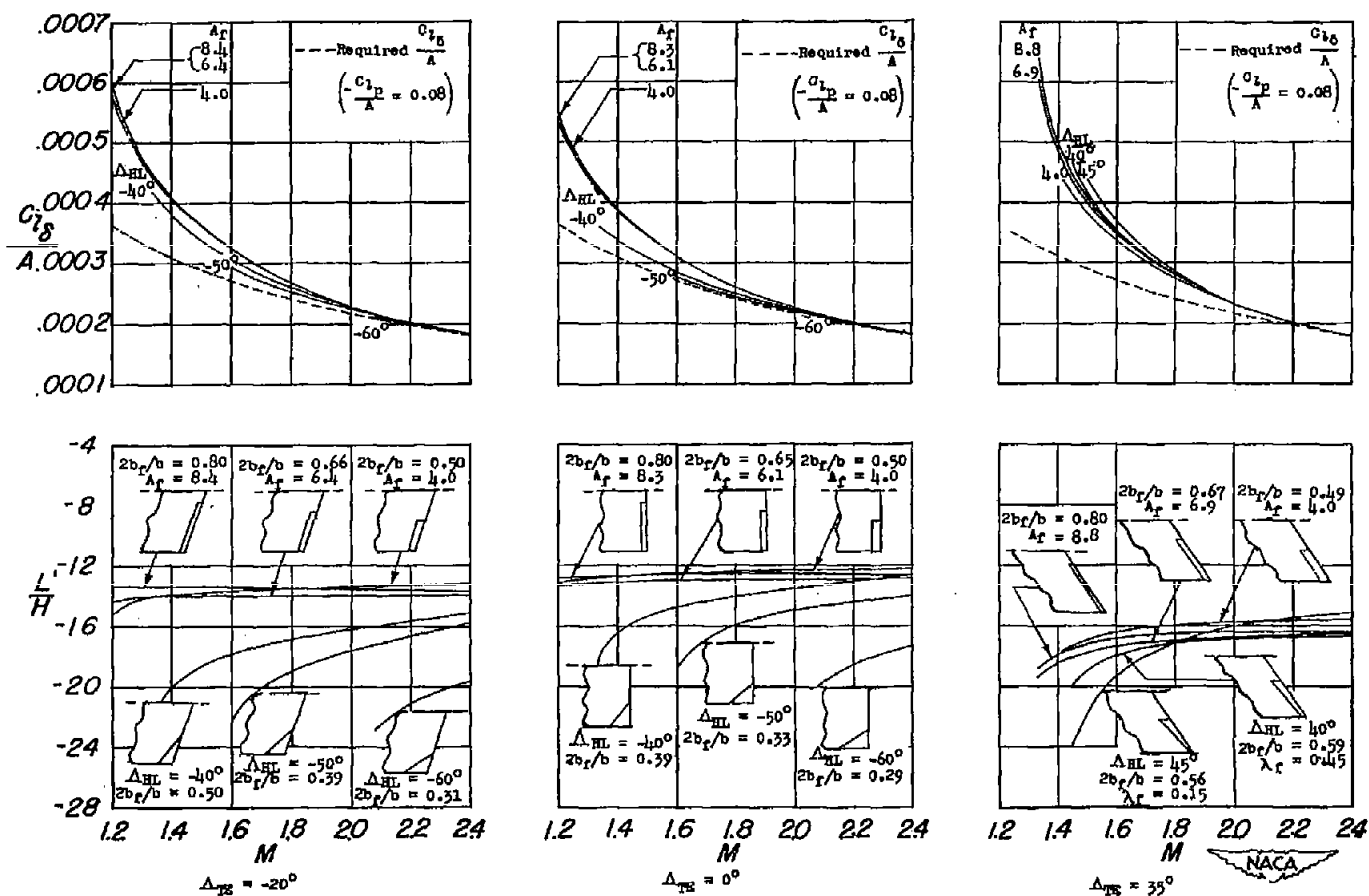
Figure 13.- Rolling-moment and hinge-moment parameters of various control plan forms on example wings having sweptforward, unswept, and sweptback trailing edges and having damping coefficients of $-\dfrac{C_{l_p}}{A} = 0.08$.
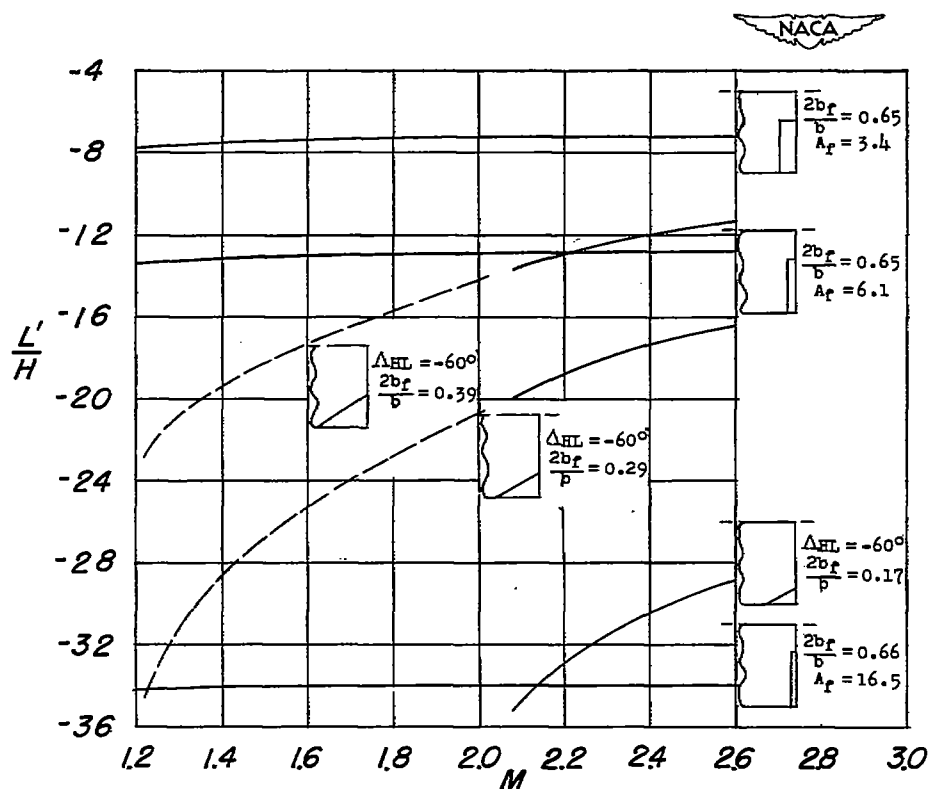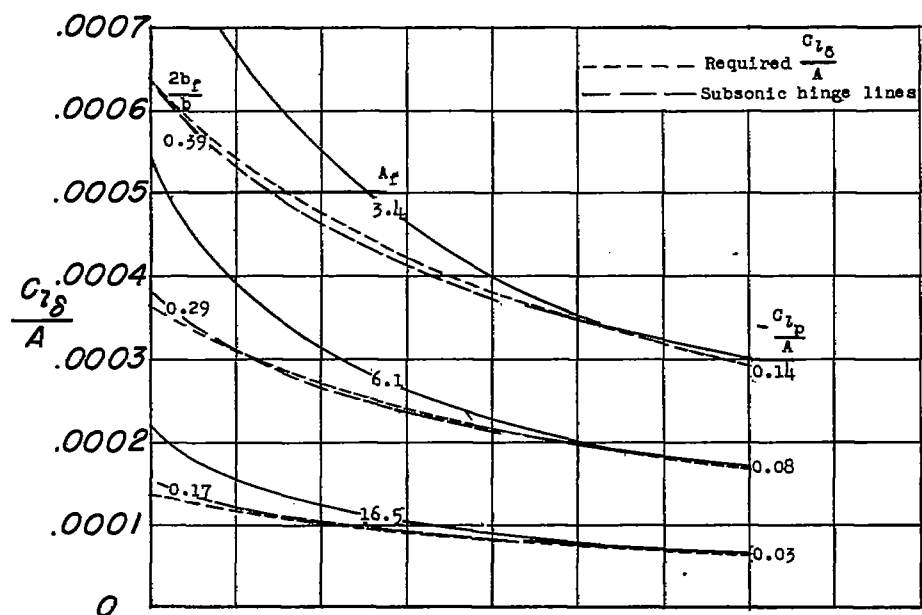
Figure 14.- Rolling-moment and hinge-moment parameters of various control plan forms on wings having unswept trailing edges and having damping coefficients $-\dfrac{C_{l_p}}{A}$ of 0.03, 0.08, and 0.14.
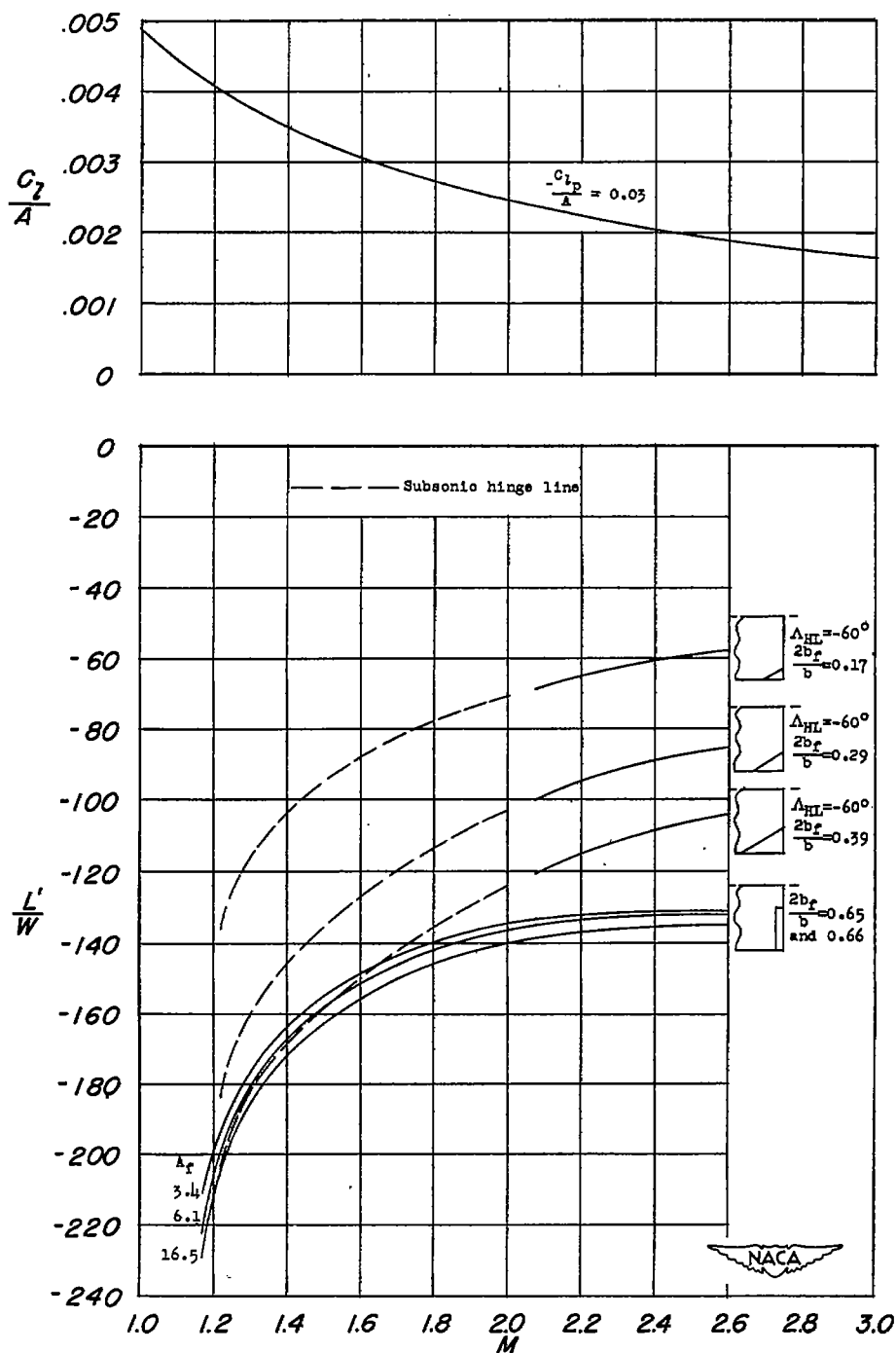
Figure 15.- Required rolling moments and ratios of rolling moment to deflection work for various control plan forms on an example wing having an unswept trailing edge and a damping coefficient $-\dfrac{C_{l_p}}{A}$ of 0.03.